Mathematics HL/SL

Graphic display calculators
First examinations 2006

Diploma Programme

Teacher support material

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Mathematics HL/SL: graphic display calculators—teacher support material

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Introduction

Purpose of this teacher support material

This teacher support material (TSM) is intended to support teachers using graphic display calculators (GDC) in Diploma Programme (DP) mathematics higher level (HL), mathematics standard level (SL) and further mathematics SL courses. It may also prove helpful to support technology use in other subject areas.

There has been much discussion about GDC use, and it is hoped that this TSM will address some of the issues and questions raised by teachers and students. These include the following.

- How to use the calculator
- How to use the calculator to enhance teaching and learning
- What students should write down in examinations when they have used a calculator
- How to manage the calculator memory

These issues are common to all subjects that permit the use of calculators in examinations and the information is relevant to any teacher using technology in teaching. Detailed examples have been included to illustrate the points discussed and more examples can be found on the online curriculum centre (OCC). It is hoped that teachers will add to these suggestions by visiting the discussion forum for calculators.

GDC background

The IBO first allowed the use of GDCs in the mid-1990s and the GDC became compulsory in three of the DP mathematics courses in September 1998. It was quickly realized that this decision would have a more significant effect on the teaching and assessment of mathematics than had been originally anticipated. The first problem was the wide disparity in the capabilities of the various models that students were using. As a result, it was decided to produce minimum requirements for the GDC and these were announced to schools in August 1999.

From 2001 onwards, students using only four-function scientific calculators or early versions of the GDC were at a disadvantage in examinations. Examiners set questions assuming that all students had a GDC with the minimum functionalities.

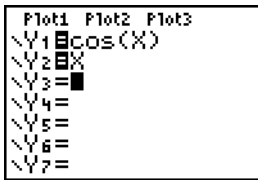
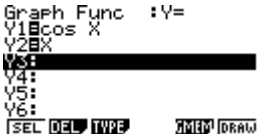
In 2004 the GDC became compulsory in all DP mathematics courses. Students taking any mathematics examinations from May 2006 onwards will be at a disadvantage if they do not have access to a suitable GDC. The *Vade Mecum* describes conditions of use in examinations and includes a list of recommended/approved models.

GDC models used in this TSM

Data collected suggests that new models are acquired and gain significant hold within only a few years of introduction into the marketplace. By the end of 2004, most students taking mathematics HL examinations had access to one of the models listed below.

Texas Instruments	Casio
TI-84 Plus Silver Edition	CFX-9850G Plus/Graph 65 Plus
TI-84 Plus	FX1.0 Plus
TI-83 Plus Silver Edition	CFX-9950 Plus
TI-83 Plus	FX-9750G Plus/Graph 35 Plus

Instructions provided in this document are split into two columns representing the two main models used: the Texas Instruments TI-84 Plus Silver Edition (TI-84+SE) on the left, and the Casio CFX-9850 Plus (Casio 9850+) on the right. For each model, the instructions are divided into a further two columns. The left column displays the buttons that should be pressed ("Press") and the right column displays the screen shots that should appear after following the instructions ("Result"). See below for an example of this layout.

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	Y= Remove any stored graphs. cos X,T,θ,n) ENTER X,T,θ,n ENTER		MENU 5 Remove any stored graphs. cos X,θ,T EXE X,θ,T EXE	

Please note that the screen shots shown in the instructions may differ depending on the user settings and the operating system installed on the calculator.

Most examples displayed under the heading of the TI-84+SE are accessible on a model from the basic TI-83 Plus upwards. The TI-83 (non-plus) is practically obsolete and cannot store or run flash (ROM) applications (Apps) and functionality upgrade is limited to a number of add-on programs.

Apart from one or two key variations in text and colour functionalities, the Casio 9850+ is functionally identical to the Casio FX-9750 Plus, the Casio Graph 65 Plus and the Casio Graph 35 Plus. Functionalities found on the Casio 9850+ are also accessible on the FX1.0 Plus and the Casio CFX-9950 Plus, although the function keys and menu numbering may differ between these models. Non-plus versions of Casio calculators are practically obsolete and might not meet minimum requirements for some subjects.

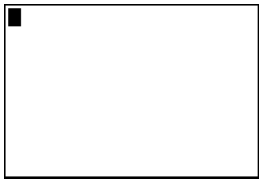
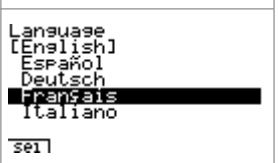
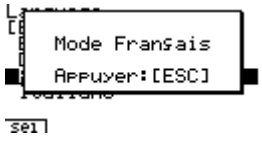
Please note that there may be alternative ways to achieve the same results. The approaches demonstrated here are not necessarily the best or the simplest of the options available.

GDC language features

Language options have become an important feature of recent GDC models. Texas Instruments now packages 12 "language localizers" with its new models and Casio offers a number of "language add-ons" for the FX1.0 Plus. Running a language localizer (TI) or language add-on (Casio) changes the language of most GDC menus, but with some limitations and exceptions. No language features are available on the Casio 9850+. The Casio Graph 35 and Graph 65 are marketed to the French language markets. Functionality is identical to the Casio 9850+; the only difference is that some keys have French and English text.

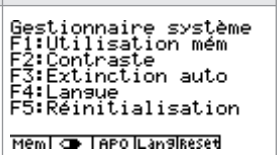
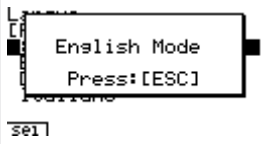
The French version of this TSM uses French language screen shots from the TI-84+SE (with the flash Apps "Français" installed). The Spanish version uses Spanish language screen shots from the TI-84+ SE (with the flash Apps "Spanish" installed). A Chinese version of the flash Apps Catalog Help is available for users with a preference for help in Chinese. See below for instructions.

Example 1: Setting the menu language to French

	TI-84+SE		Casio FX1.0 Plus (feature not available on 9850+)	
	Press	**Result**	**Press**	**Result**
1.	APPS ALPHA COS to locate “Français”.		MENU ALPHA ln to access System Manager.	
2.	ENTER to run “Français”.		F4 to access language menu.	
3.	1 to set menu language to French.		▼ ▼ to select “Français”.	
4.			EXE to set language mode to French.	

The same method is used to set the menu language to one of the other available languages.

Example 2: Setting the menu language back to English

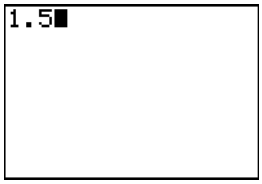
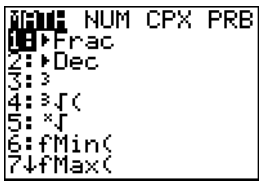
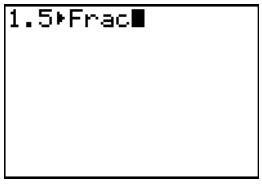
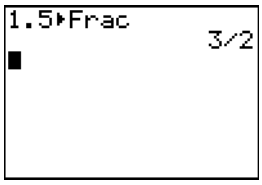
	TI-84+SE		Casio FX1.0 Plus (feature not available on 9850+)	
	Press	**Result**	**Press**	**Result**
1.	APPS ALPHA COS to locate “Français”.		MENU ALPHA ln to access System Manager.	
2.	ENTER to run “Français”.		F4 to access language menu.	
3.	2 to set menu language to English.		EXE to set language mode to English.	

Catalog Help

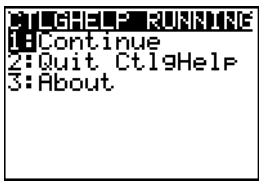
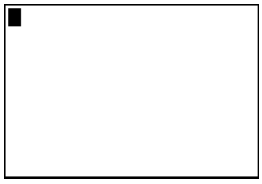
Example 1: Installing Catalog Help

	TI-84+SE	
	Press	**Result**
1.	APPS ALPHA PRGM Use the arrow keys to locate Ctlg Help.	
2.	ENTER to install Catalog Help, then any key. ENTER to return to the home screen.	

Example 2: Using Catalog Help

	TI-84+SE	
	Press	**Result**
1.	1 • 5	
2.	MATH to access Math menu. Frac function is selected.	
3.	+ to run Ctlg Help.	
4.	TRACE to paste the Frac function into the home screen.	
5.	ENTER to apply.	

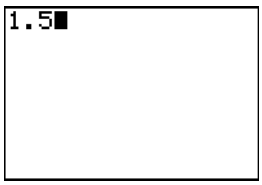
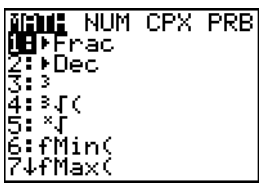
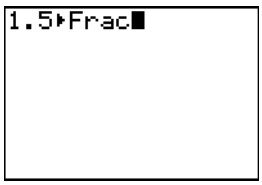
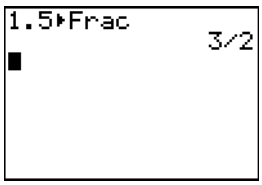
Example 3: Quitting Catalog Help

	TI-84+SE	
	Press	**Result**
1.	APPS ALPHA PRGM Use the arrow keys to locate Ctlg Help.	
2.	ENTER to run Catalog Help.	
3.	2 to quit Catalog Help.	

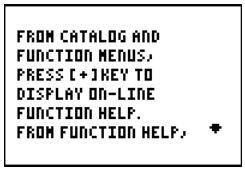
Example 4: Installing Chinese Catalog Help

	TI-84+SE	
	Press	**Result**
1.	APPS ALPHA PRGM Use the arrow keys to locate Chinese Catalog Help.	
2.	ENTER to install Chinese Catalog Help, then any key.	

Example 5: Using Chinese Catalog Help

	TI-84+SE	
	Press	**Result**
1.	1 • 5	
2.	MATH to locate Math menu. Frac function is selected.	
3.	+ to run Catalog Help. Chinese help screen will appear.	
4.	ENTER ENTER to paste the Frac function into the home screen.	
5.	ENTER to apply.	

Example 6: Quitting Chinese Catalog Help

	TI-84+SE	
	Press	**Result**
1.	APPS ALPHA PRGM Use the arrow keys to locate Ctlg Help.	
2.	ENTER to run Catalog Help. 2 to quit Catalog Help.	

Using the TI-84 Plus SE and the Casio CFX-9850 Plus

In this section, step-by-step examples of selected GDC features, functions and tools are displayed. They are organized by function task and often use examples taken from previous examination papers or specimen papers.

This TSM assumes some basic skills in working with the GDC. New GDC users should get acquainted with their calculator by first referring to the "Getting Started" chapter in the Texas Instruments manual, or to the "Quick-Start" section of the Casio "Owner's Manual". Online manuals are also available from the web sites listed below.

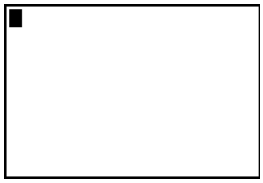
Particular attention should be paid to the instructions about entering expressions, editing, deleting, using parentheses, storing values, changing mode settings (especially settings in degrees versus radians), setting the graph window, editing tables, graphs, lists and matrices, performing simple calculations, plotting data, resetting defaults, installing and running applications and understanding error messages.

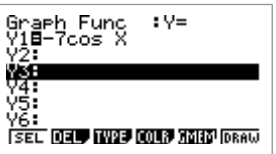
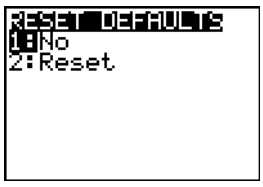
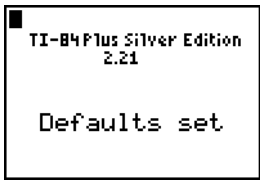
Texas Instruments	http://education.ti.com/us/global/guides.html
Casio	http://www.casio.co.jp/edu_e/support/ http://www.silrun.info/
Sharp	http://www.sharpusa.com/products/support/0,2309,,00.html
Hewlett Packard	http://www.hpmuseum.org/software/swcd.htm

Basic graphing skills

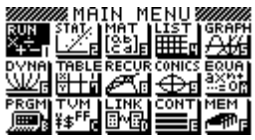
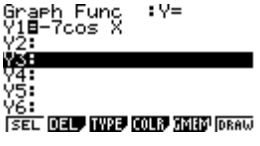
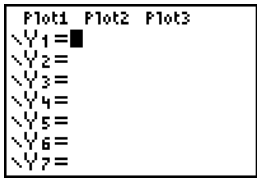
Choosing an appropriate window

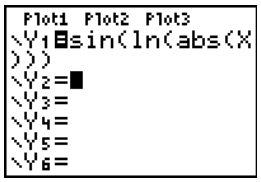
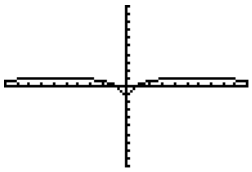
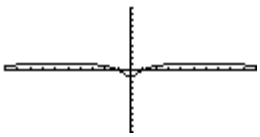
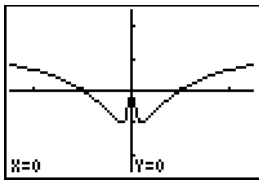
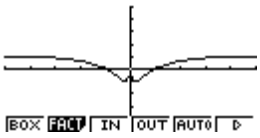
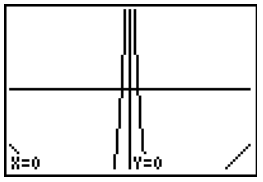
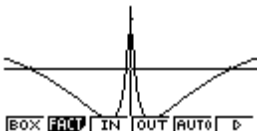
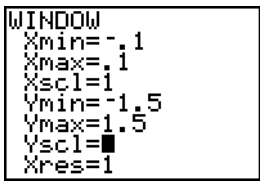
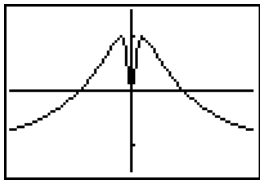
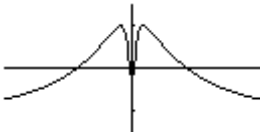
Task A: Reset defaults (graphing)

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	From the home screen.		MENU	
2.	2nd + to access memory functions. (Note: May need to use CLEAR or 2 MODE to get to the home screen.)		Use the arrow keys to highlight GRAPH.	

3.	7 to activate Reset.		EXE to activate the graph function.	
4.	2 to select Reset Defaults.		SHIFT F3 to access V-Window (the View Window setting).	
5.	2 to confirm.		F3 to standardize the View Window setting.	
	The default graph window on the TI-84+SE is ± 10 on both axes.		The standard graph window on the Casio CFX-9850 + is ± 10 on both axes.	

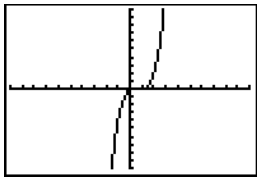
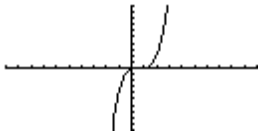
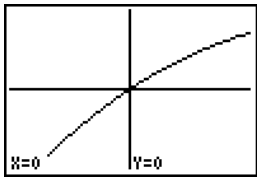
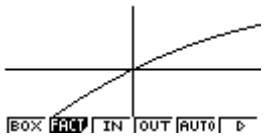
Task B: Using the GDC to graph the function, $y = \sin(\ln|x|)$, demonstrate the difference the window makes to the appearance of a graph

	TI-84+SE		Casio 9850+	
	Use default graph window ± 10.		Use standard graph window ± 10.	
	Press	Result	Press	Result
1.	From the home screen. (Note: May need to use CLEAR or 2 MODE to get to home screen.)		MENU	
2.			Use the arrow keys to highlight GRAPH.	
3.			EXE to activate the graph function.	
4.	Y= Use the arrow keys to highlight any stored graphs, and CLEAR to remove them.		Use the arrow keys to highlight any stored graphs, and F2 then F1 to remove them.	

5.	SIN LN MATH) 1 X,T,θ,n))) ENTER		sin (ln (OPTN F5 F1 (X,θ,T)) EXE	
6.	GRAPH		F6	
7.	ZOOM 2 ENTER		SHIFT F2 F3	
8.	ENTER		F3	
It appears with each successive zoom that a different graph is displayed. This is even more apparent when, instead of using zoom, the window is changed to $y = -1.5$ to +1.5 (and kept at that), while x is changed from –2 to +2, –1 to +1 and –0.1 to +0.1.				
9.	Change the window settings to: Xmin = –0.1 Xmax = 0.1 Xscl = 1 Ymin = –1.5 Ymax = 1.5 Yscl = 1.		Change the window settings to: Xmin : –0.1 max : 0.1 scale : 1 Ymin : –1.5 max : 1.5 scale : 1.	
	WINDOW (–) • 1 ENTER • 1 ENTER 1 ENTER (–) 1 • 5 ENTER 1 • 5 ENTER 1 ENTER		SHIFT F2 to access V-Window. (–) • 1 EXE • 1 EXE 1 EXE (–) 1 • 5 EXE 1 • 5 EXE 1 EXE Use EXE to return to Graph window.	
10.	GRAPH		F6	

On the default/standard graph window the graph of $y = x^3 - 2x^2 + x$ appears to intersect the x-axis only once. By using zoom or changing the window settings it is possible to observe other key features of the curve.

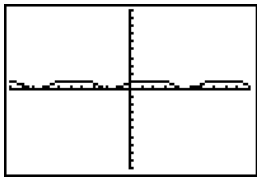
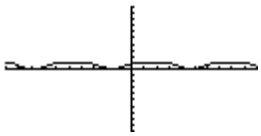
Task C: Graph the function $y = x^3 - 2x^2 + x$ in the default/standard window

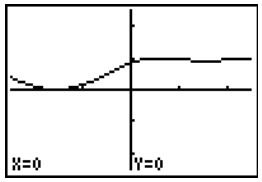
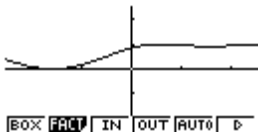
	TI-83/84+SE		Casio 9850+	
	Use default graph window ± 10.		Use standard graph window ± 10.	
	Press	Result	Press	Result
1.	Y= Remove any stored graphs. Graph the function $y = x^3 - 2x^2 + x$. X,T,θ,n ^ 3 − 2 X,T,θ,n ^ 2 + X,T,θ,n ENTER GRAPH		MENU 5 Remove any stored graphs. Graph the function $y = x^3 - 2x^2 + x$. X,θ,T ^ 3 − 2 X,θ,T ^ 2 + X,θ,T EXE F6	
2.	ZOOM 2 ENTER ENTER ENTER to zoom in.		SHIFT F2 F3 F3 F3 F3 F3 F3 to zoom in.	

It is possible to observe that the graph intersects the x-axis at least once near or at the origin by zooming in with iterative use of the zoom function.

An alternative would be to change the window settings, with x from –1 to +2 and y from –0.5 to +0.5.

Task D: On the default screen, the graph $y = \sin(1 + \sin x)$ appears to have a local maximum between $x = 0$ and $x = 4$. Use zoom and change the window settings to reveal further features of this curve

	TI-84+SE		Casio 9850+	
	Use default graph window ± 10.		Use standard graph window ± 10.	
	Press	Result	Press	Result
1.	Y= Remove any stored graphs. Graph the function $y = \sin(1 + \sin x)$. SIN 1 + SIN X,T,θ,n)) ENTER GRAPH		MENU 5 Remove any stored graphs. Graph the function $y = \sin(1 + \sin x)$. sin (1 + sin (X,θ,T)) EXE F6	

2.	ZOOM 2 ENTER to zoom in.		SHIFT F2 F3 F3 to zoom in.	

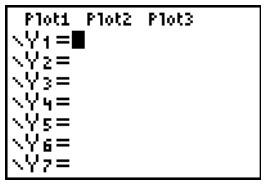
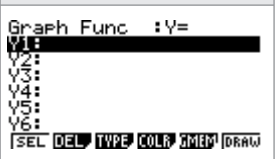
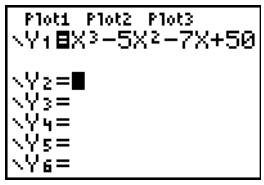
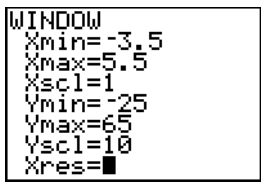
A local minimum value between two local maximums can be observed by using the zoom function or by changing the window settings for x from –0.5 to +4, and for y from 0 to +1.5.

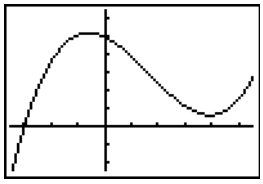
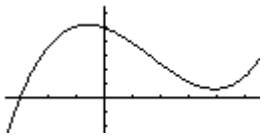
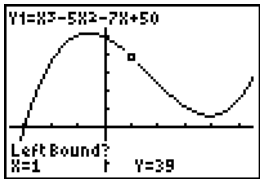
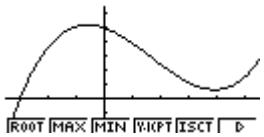
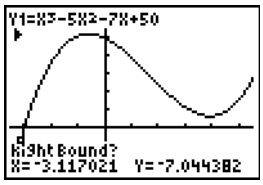
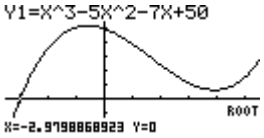
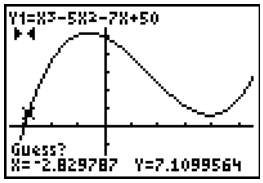
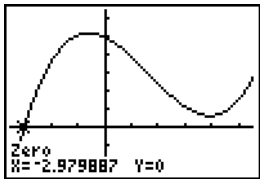
Finding zeros, maxima and minima

Once the graph of a function has been obtained, the GDC can be used to determine many important features of the graph.

Example: Consider the function $f(x) = x^3 - 5x^2 - 7x + 50$.

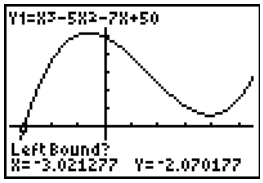
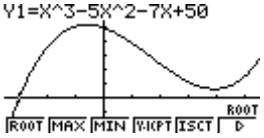
Task A: Find the coordinates of the zero

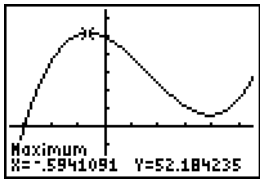
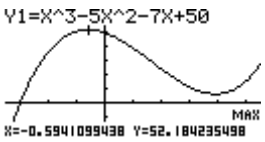
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	In Y1 enter $f(x)$. X,T,θ,n MATH 3 (–) 5 X,T,θ,n x^2 (–) 7 X,T,θ,n + 5 0 ENTER		In Y1 enter $f(x)$. X,θ,T ^ 3 (–) 5 X,θ,T ^ 2 (–) 7 X,θ,T + 5 0 EXE	
3.	Change the window settings to: Xmin = –3.5 Xmax = 5.5 Xscl = 1 Ymin = –25 Ymax = 65 Yscl = 10.		Change the window settings to: Xmin : –3.5 max : 5.5 scale : 1 Ymin : –25 max : 65 scale : 10.	
	WINDOW (–) 3 • 5 ENTER 5 • 5 ENTER 1 ENTER (–) 2 5 ENTER 6 5 ENTER 1 0 ENTER		SHIFT F3 (–) 3 • 5 EXE 5 • 5 EXE 1 EXE (–) 2 5 EXE 6 5 EXE 1 0 EXE	

4.	GRAPH		EXE F6	
5.	2nd TRACE 2 to find the zero.		SHIFT F5	
6.	◄ to move cursor just to the left of the zero, then ENTER.		F1 to find the zero.	
7.	A cross appears on the curve at that point as well as a marker at the top of the screen. At the bottom of the screen there is a prompt for the right bound. Use the arrow keys to move cursor just to the right of the zero, then ENTER.		Note: A powerful feature of G-Solve is that further roots, maxima and minima can be found by using the left and right arrow keys.	
8.	Another cross appears on the curve at that point as well as another marker above the curve. At the bottom of the screen "Guess?" appears. Use the arrow keys to move cursor as close as possible to the zero, then ENTER.			

The zero is at (–2.979887, 0).

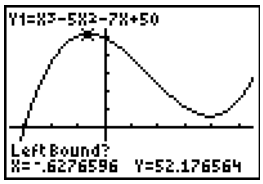
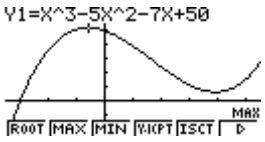
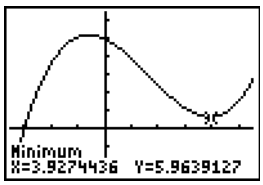
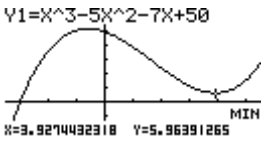
Task B: Find the coordinates of the local maximum

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	2nd TRACE 4 to find the maximum.		SHIFT F5	

2.	To find the zero, use the arrow keys to move the cursor first just to the left of the maximum, then ENTER, then just to the right of the maximum, then ENTER, and finally very near to the maximum, then ENTER. Crosses appear on the curve at the chosen points and left and right markers appear above the curve.		F2 to find the maximum.	

Below the curve the coordinates of the maximum are given. The maximum occurs at (–0.5941091, 52.184235).

Task C: Find the coordinates of the local minimum

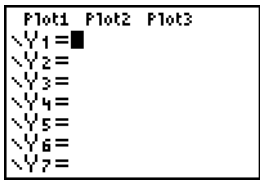
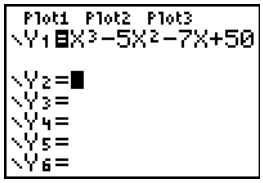
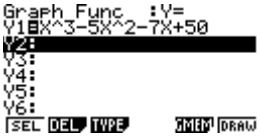
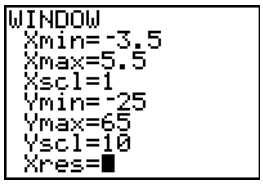
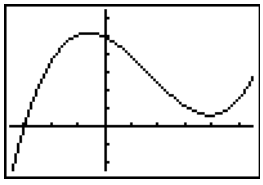
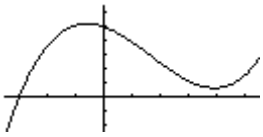
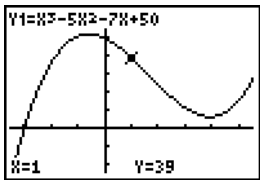
	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	2nd TRACE 3		SHIFT F5	
2.	To find the minimum, use the arrow keys to move the cursor first just to the left of the minimum, then ENTER, then just to the right of the minimum, then ENTER, and finally very near to the minimum, then ENTER.		F3 to find the minimum.	

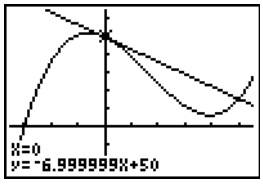
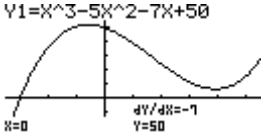
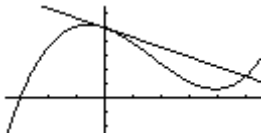
Below the curve the coordinates of the minimum are given. The minimum occurs at (3.9274436, 5.9639127).

Finding equations of tangents

Example: Consider the function $f(x) = x^3 - 5x^2 - 7x + 50$.

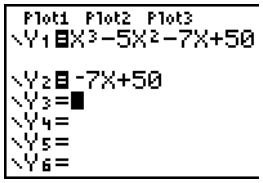
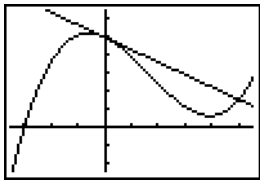
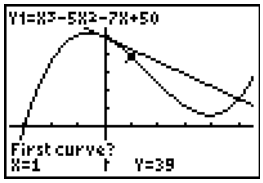
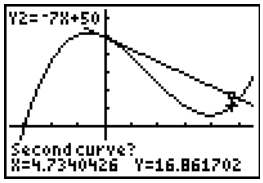
Task A: Draw the tangent to the curve at $x = 0$

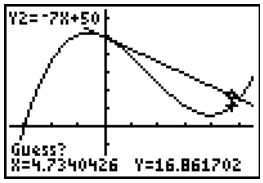
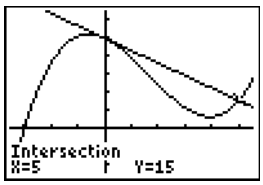
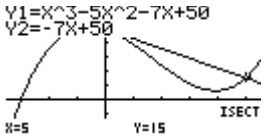
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	In Y1 enter $f(x)$. X,T,θ,n MATH 3 (−) 5 X,T,θ,n x^2 (−) 7 X,T,θ,n + 5 0 ENTER		In Y1 enter $f(x)$. X,θ,T ^ 3 (−) 5 X,θ,T ^ 2 (−) 7 X,θ,T + 5 0 EXE	
3.	Change the window settings to: Xmin = –3.5 Xmax = 5.5 Xscl = 1 Ymin = –25 Ymax = 65 Yscl = 10.		Change the window settings to: Xmin : –3.5 max : 5.5 scale : 1 Ymin : –25 max : 65 scale : 10.	
	WINDOW (−) 3 • 5 ENTER 5 • 5 ENTER 1 ENTER (−) 2 5 ENTER 6 5 ENTER 1 0 ENTER		SHIFT F3 (−) 3 • 5 EXE 5 • 5 EXE 1 EXE (−) 2 5 EXE 6 5 EXE 1 0 EXE	
4.	GRAPH		EXE F6	
5.	2nd PRGM 5 The curve is shown again with the x and y coordinates at the bottom of the screen.		SHIFT MENU ▼ ▼ ▼ ▼ F1	

6.	0 ENTER The tangent at $x = 0$ is drawn and below the graph is the equation of the tangent $y = -6.999999x + 50$		Please note: the Casio 9850+ cannot write the equation of the tangent at a point on the curve. It will give the gradient for a given value of x and the equation then has to be worked out manually. EXIT F6 SHIFT F1 ◀ ◀ ◀ ◀ EXE	
7.			EXIT ▼ (−) 7 X,θ,T + 5 0 EXE F6	

The equation of the tangent at $x = 0$ is $y = -7x + 50$.

Task B: Find the other point of intersection of this tangent with the curve

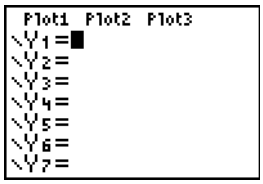
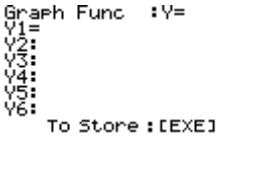
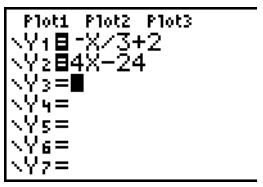
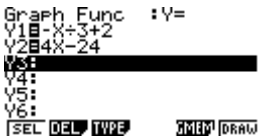
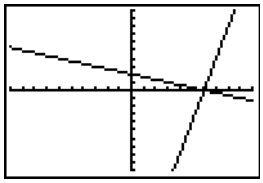
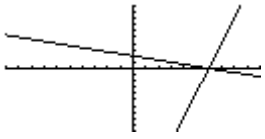
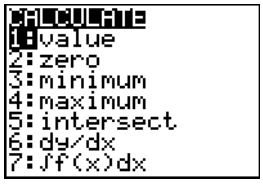
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= ▶ ▶ (−) 7 X,T,θ,n + 5 0 ENTER This enters the equation of the tangent at $x = 0$ into the Y= menu.			
2.	GRAPH			
3.	2nd TRACE 5 to find the intersection of these lines.		SHIFT F5 to find the intersection of these lines.	
4.	Below the graph there are prompts to select the curves for which the point of intersection is required. Use the arrow keys to move the cursor just to the left of the point of intersection on the cubic curve, then ENTER.		F5 The calculator finds the first intersection at $x = 0$.	

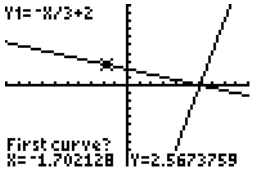
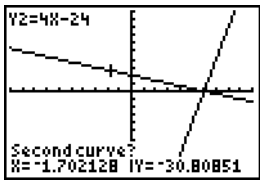
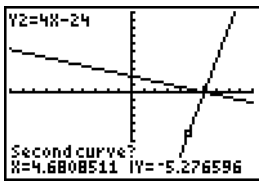
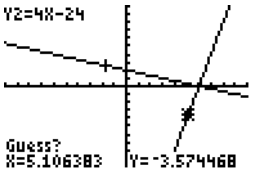
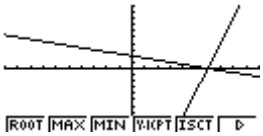
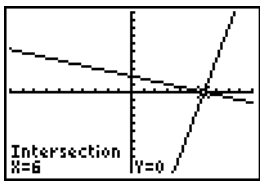
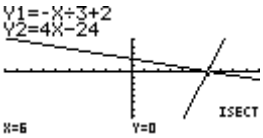
5.	A cross appears on the cubic curve at that point and there is a prompt to select the second curve. ENTER to select the tangent line as the second curve.			
6.	Move the cursor as close as possible to the intersection point, then ENTER.		▶ to get the other intersection.	

The intersection point is (5, 15).

Finding graphical solutions of equations

Task A: If possible, solve $\begin{aligned} 2x + 6y &= 12 \\ 4x - y &= 24 \end{aligned}$

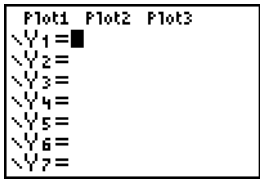
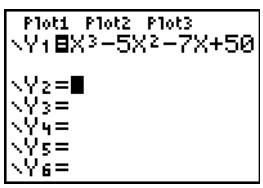
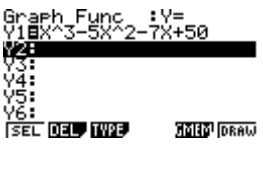
	TI-84+SE		Casio 9850+	
	Use default graph window ± 10.		Use standard graph window ± 10.	
	Press	Result	Press	Result
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	(−) X,T,θ,n ÷ 3 + 2 ENTER 4 X,T,θ,n − 2 4 ENTER		(−) X,θ,T ÷ 3 + 2 EXE 4 X,θ,T − 2 4 EXE	
3.	GRAPH		F6	
4.	2nd TRACE			

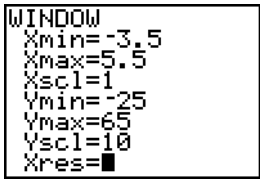
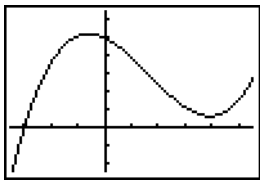
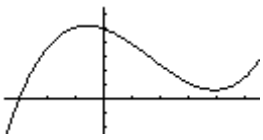
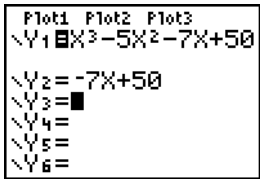
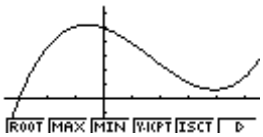
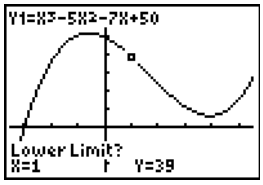
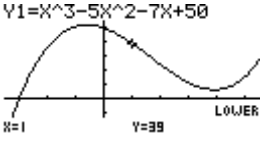
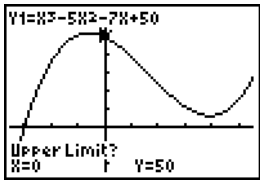
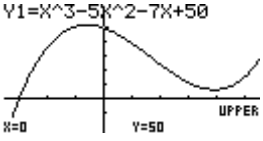
5.	5			
6.	ENTER			
7.	▶ until a star appears on screen.			
8.	ENTER		SHIFT F5	
9.	ENTER		F5	

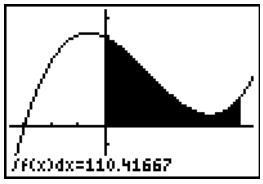
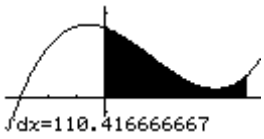
Finding the area under the curve between two points

Example: Consider the function $f(x) = x^3 - 5x^2 - 7x + 50$.

Task A: Find the area under the curve between $x = 0$ and $x = 5$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	In Y1 enter $f(x)$. X,T,θ,n MATH 3 (−) 5 X,T,θ,n x^2 (−) 7 X,T,θ,n + 5 0 ENTER		In Y1 enter $f(x)$. X,θ,T ^ 3 (−) 5 X,θ,T ^ 2 (−) 7 X,θ,T + 5 0 EXE	

3.	Change the window settings to: Xmin = –3.5 Xmax = 5.5 Xscl = 1 Ymin = –25 Ymax = 65 Yscl = 10.		Change the window settings to: Xmin : –3.5 max : 5.5 scale : 1 Ymin : –25 max : 65 scale : 10.	
	WINDOW (–) 3 • 5 ENTER 5 • 5 ENTER 1 ENTER (–) 2 5 ENTER 6 5 ENTER 1 0 ENTER		SHIFT F3 (–) 3 • 5 EXE 5 • 5 EXE 1 EXE (–) 2 5 EXE 6 5 EXE 1 0 EXE	
4.	GRAPH		EXE F6	
5.	Y= If you have just done the exercise on tangents, to remove the tangent line from the graph move the cursor over the = sign on Y2, then ENTER. The black square over the equals sign will disappear. If you have not yet done the exercise on tangents, there will be no equation in the Y2 line.		SHIFT F5	
6.	GRAPH 2nd TRACE 7 to find the area under the curve. The graph is displayed and there is a prompt to enter the lower limit for the area.		F6 F3 to find the area under the curve.	
7.	0 ENTER		The prompt asks for the lower bound. Scroll using ◀ until you get to $x = 0$, then EXE.	

8.	There is now a prompt for the upper limit for the area. 5 ENTER		There is now a prompt for the upper limit for the area. Scroll using ▶ until you get to $x = 5$, then EXE.	

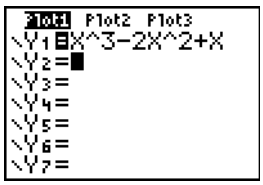
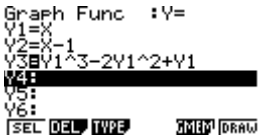
The area under the curve between zero and five is shaded and the value of the integral is given as 110.41667.

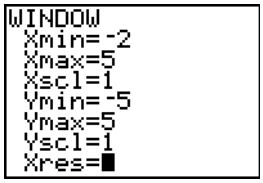
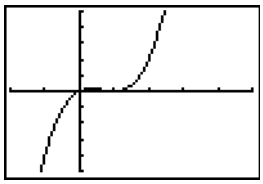
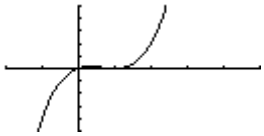
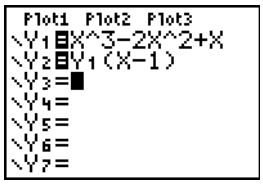
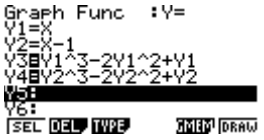
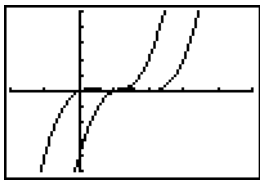
Finding transformations

Consider the curve $f(x) = x^3 - 2x^2 + x$.

Task A: Find the transformation of this curve where $g(x) = f(x-1)$

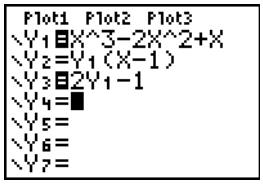
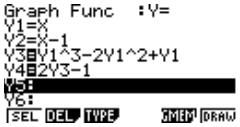
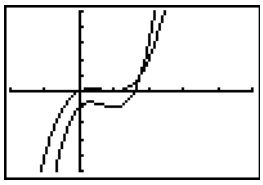
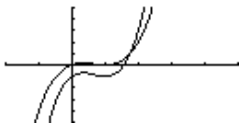
Note that the TI-84+SE and Casio 9850+ vary significantly in the syntax used.

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs. X,T,θ,n ^ 3 − 2 X,T,θ,n ^ 2 + X,T,θ,n ENTER		MENU 5 Remove any stored graphs.	
2.			Enter: Y1=X Y2=X–1 Y3=$Y1^3$–2$Y1^2$+Y1. X,θ,T EXE X,θ,T − 1 EXE VARS F4 F1 1 ^ 3 − 2 VARS F4 F1 1 ^ 2 + VARS F4 F1 1 EXE Deselect the graphs for Y1 and Y2: ▲ ▲ F1 ▲ F1.	

<table>
<tr><td>3.</td><td colspan="2">Change the window settings to:
Xmin = –2
Xmax = 5
Xscl = 1
Ymin = –5
Ymax = 5
Yscl = 1.</td><td colspan="2">Change the window settings to:
Xmin : –2
max : 5
scale : 1
Ymin : –5
max : 5
scale : 1.</td></tr>
<tr><td></td><td>WINDOW
(–) 2 ENTER
5 ENTER
1 ENTER
(–) 5 ENTER
5 ENTER
1 ENTER</td><td> 
</td><td>SHIFT F3
(–) 2 EXE
5 EXE
1 EXE
(–) 5 EXE
5 EXE
1 EXE</td><td> 
</td></tr>
<tr><td>4.</td><td>GRAPH</td><td>  </td><td>EXE F6</td><td>  </td></tr>
<tr><td>5.</td><td>Y= ◀
VARS ▶ 1 ENTER
(X,T,θ,n – 1
) ENTER</td><td> 
</td><td>EXIT
Then ▼ ▼ ▼ to get to Y4 and:
VARS F4 F1 2
^ 3 – 2
VARS F4 F1 2
^ 2 +
VARS F4 F1 2 EXE.</td><td> 
</td></tr>
<tr><td>6.</td><td>GRAPH</td><td>  </td><td>F6</td><td>  </td></tr>
</table>

The graph now shows a translation to the right of one unit.

Task B: Find the transformation of this curve where $h(x) = 2f(x) - 1$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Use the arrow keys to scroll down to Y3. 2 VARS ▸ 1 ENTER (−) 1 ENTER ▲ ▲ ◀ ENTER to remove the graph for Y2.		EXIT ▲ F2 F1 to remove the graph for Y4. Then enter $2f(x) - 1$ into Y4. 2 VARS F4 F1 3 (−) 1 EXE	
2.	GRAPH		F6	

The graph now shows a stretch of scale factor two in the *y* direction and a translation of minus one also in the *y* direction.

Selected basic tools

Using the polynomial root finder

Many equations can be solved most efficiently by reducing the problem to one of finding the zero or root of a graph. However, there may be situations where a more direct approach is preferred. The following examples illustrate how the polynomial root finder can be used.

Task A: Solve $x^3 - 2x^2 - 5x + 6 = 0$ **for** x

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	APPS and then use the arrow keys to highlight PolySmlt. ENTER and then any key to access the PolySmlt submenu. Alternatively, APPS ALPHA 8 ENTER to jump to the PolySmlt submenu, then any key.	MAIN MENU 1:Poly Root Finder 2:Simult Eqn Solver 3:About 4:Poly Help 5:Simult Help 6:Quit PolySmlt	Use the arrow keys to highlight EQUA and then EXE to access the Equation submenu. Alternatively, ALPHA X,θ,T to access the Equation submenu.	Equation Select Type F1:Simultaneous F2:Polynomial F3:Solver SIML POLY SOLV
2.	ENTER to run the polynomial root finder.	POLY ROOT FINDER Degree of Poly = MAIN LOAD	F2 to run the polynomial root finder.	Polynomial Data For 3 Degree In Memory Degree? 2 3
3.	3 ENTER to enter a degree 3 polynomial.	a3x^3+...+a1x+a0=0 a3 = a2 = a1 = a0 = MAIN DEGR CLR LOAD SOLVE	F2 to choose degree 3 polynomial.	aX³+bX²+cX+d=0 a b c d 0 0 0 0 0 SOLV DEL CLR
4.	Enter coefficients. 1 ENTER (−) 2 ENTER (−) 5 ENTER 6 ENTER	a3x^3+...+a1x+a0=0 a3 =1 a2 =-2 a1 =-5 a0 = MAIN DEGR CLR LOAD SOLVE	Enter coefficients. 1 EXE (−) 2 EXE (−) 5 EXE 6 EXE	aX³+bX²+cX+d=0 a b c d 1 -2 -5 6 6 SOLV DEL CLR
5.	GRAPH to solve.	a3x^3+...+a1x+a0=0 x1 =1 x2 =-2 x3 =3 MAIN COEFS STOa STOx STOy	F1 to solve.	aX³+bX²+cX+d=0 X 1 -2 3 REPT

Task B: Solve $0 = x^4 - 3x^3 - 3x^2 + 11x - 6$ **for** x

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	APPS and then use the arrow keys to highlight PolySmlt. ENTER and then any key to access the PolySmlt submenu. Alternatively, ALPHA 8 ◀ ◀ ENTER to jump to the PolySmlt submenu, then any key.	POLYNOMIAL ROOT FINDER Degree of Poly = MAIN | | | LOAD	On the 9850+, the polynomial root finder is limited to degree 2 or 3. The example below makes use of the equation solver to achieve similar results.	
2.	ENTER to run the polynomial root finder.	POLYNOMIAL ROOT FINDER Degree of Poly = MAIN | | | LOAD		
3.	4 ENTER 1 ENTER (−) 3 ENTER (−) 3 ENTER 1 1 ENTER (−) 6 ENTER	a4x^4+...+a1x+a0=0 a4 =1 a3 =-3 a2 =-3 a1 =11 a0 =6 MAIN | DEGR | CLR | LOAD | SOLVE	ALPHA X,θ,T	Equation Select Type F1:Simultaneous F2:Polynomial F3:Solver SIML POLY SOLV
4.	GRAPH	a4x^4+...+a1x+a0=0 x1 =.9999995934 x2 =1.000000407 x3 =3 x4 =-2 MAIN | COEFS | STOa | STOx | STOy	F3 to run the equation solver, and 0 SHIFT • ALPHA X,θ,T ALPHA ÷ ^ 4 + ALPHA log ALPHA ÷ ^ 3 + ALPHA ln ALPHA ÷ ^ 2 + ALPHA sin ALPHA ÷ + ALPHA cos EXE 1 EXE (−) 2 EXE (−) 3 EXE (−) 3 EXE 1 1 EXE (−) 6 EXE .	Eq:0=AT^4+BT^3+CT^2+ A=1 T=-2 B=-3 C=-3 D=11 E=-6 RCL DEL SOLV
5.	Note that the polynomial root finder did not find the exact roots, but only approximations.		▲ or ▼ to highlight T.	Eq:0=AT^4+BT^3+CT^2+ A=1 T=-2 B=-3 C=-3 D=11 E=-6 RCL DEL SOLV

6.			F6 to solve.	Eq:0=AT^4+BT^3+CT^2+ T=-2 Lft=0 Rgt=0 REPT
7.			Repeat these steps but with alternative guess values for T to find additional roots.	Eq:0=AT^4+BT^3+CT^2+ T=3 Lft=0 Rgt=0 REPT

Using the equation solver

Many equations can be solved most efficiently by reducing the problem to one of finding the point of intersection of two graphs. However, there may be situations where a more direct approach is preferred. The following two examples illustrate how the equation solver can be used.

Task A: Solve $e^x = x^3$, for x between 4 and 5

	TI-84+SE		Casio 9850+	
	The equation must first be rewritten in the form $0 = f(x)$. $0 = e^x - x^3$			
	Press	**Result**	**Press**	**Result**
1.	MATH 0 ◂ CLEAR	EQUATION SOLVER eqn:0= [A]	MENU 1 OPTN F4 F1	Solve(Solve d/dx ∫dx ∫'dx ▷
2.	2nd LN X,T,θ,n) − X,T,θ,n MATH 3 ENTER	e^(X)−X³=0 X= bound={-1E99,1…	SHIFT ln X,θ,T − X,θ,T ^ 3 , 4 • 5 , 4 , 5)	Solve(eX-X^3,4.5,4,5) Solve d/dx ∫dx ∫'dx ▷
3.	4 • 5 ALPHA ENTER	e^(X)−X³=0 ▪X=.5364036549… bound={-1E99,1… ▪left-rt=0	EXE	Solve(eX-X^3,4.5,4,5) 4.536403655 Solve d/dx ∫dx ∫'dx ▷
	The solution is 4.5364036549…		The solution is 4.536403655…	

Note that only an approximation is given and the accuracy may vary depending on calculator settings.

Task B: **Solve** $\sin^2(x) = 2\cos(3x-1)$, **for** x **between** 0 **and** $\frac{\pi}{2}$

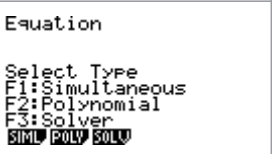
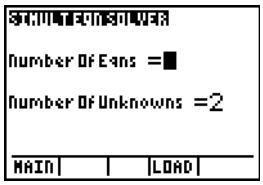
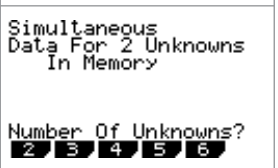
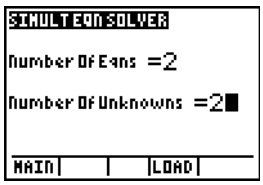
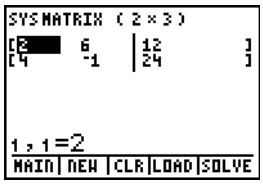
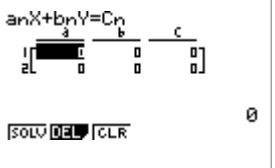
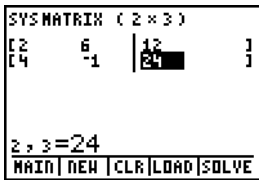
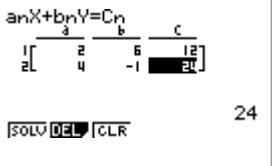
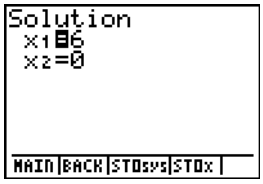
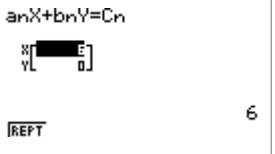
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	MATH 0 ▬ CLEAR	EQUATION SOLVER eqn:0= Solve d/dx ∫dx	MENU 1 OPTN F4 F1	Solve(Solve d/dx ∫dx
2.	(SIN X,T,θ,n)) x^2 − 2 cos 3 X,T,θ,n) + 1 ENTER	(sin(X))²-2co…=0 X=.5364036549… bound={-1E99,1…	(sin (X,θ,T)) x^2 − 2 cos (3 X,θ,T) + 1 , SHIFT EXP ÷ 4 , 0 , SHIFT EXP ÷ 2)	Solve((sin (X))²-2cos (3X)+1,π÷4,0,π÷2) Solve d/dx ∫dx
3.	2nd ^ ÷ 4 ENTER	(sin(X))²-2co…=0 X=.78539816339… bound={-1E99,1…	EXE	Solve((sin (X))²-2cos (3X)+1,π÷4,0,π÷2) 0.3286437467 Solve d/dx ∫dx
4.	2nd (0 , 2nd ^ ÷ 2 2nd) ENTER	(sin(X))²-2co…=0 X=.78539816339… bound={0,1.570…		
5.	▬ ALPHA ENTER	(sin(X))²-2co…=0 ▪X=.32864374674… bound={0,1.570… ▪left-rt=0		

The solution is 0.32864374674…

Using simultaneous equation solvers

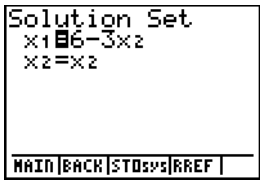
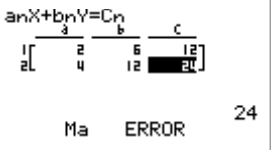
Systems of equations can be solved most efficiently by using the simultaneous equation solver.

Task A: Find a solution to the system of equations $\begin{aligned} 2x + 6y &= 12 \\ 4x - y &= 24 \end{aligned}$

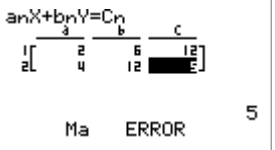
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	APPS ALPHA 8 and use the arrow keys to highlight PolySmlt.		MENU X,θ,T to access Equation submenu.	
2.	ENTER then any key to access the PolySmlt submenu.			
3.	2 to run the simultaneous equation solver.		F1 to run the simultaneous equation solver.	
4.	2 ENTER 2 to enter two equations and two unknowns.			
5.	ENTER displays coefficient menu.		F1 to select two unknowns. (Two equations are automatically selected.)	
6.	2 ENTER 6 ENTER 1 2 ENTER 4 ENTER (−) 1 ENTER 2 4 ENTER		2 EXE 6 EXE 1 2 EXE 4 EXE (−) 1 EXE 2 4 EXE	
7.	GRAPH to solve system.		F1 to solve system	

(Systems with many or no solutions may present with unexpected results on the GDC.)

Task B: Find a solution to the system of equations $\begin{matrix} 2x+6y=12 \\ 4x+12y=24 \end{matrix}$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Repeat the steps from Task A using these coefficients: 2, 6, 12, 4, 12, 24.		Repeat the steps from Task A using these coefficients: 2, 6, 12, 4, 12, 24.	
2.	Result of the simultaneous equation solver.		Result of the simultaneous equation solver.	

Task C: Find a solution to the system of equations $\begin{matrix} 2x+6y=12 \\ 4x+12y=12 \end{matrix}$

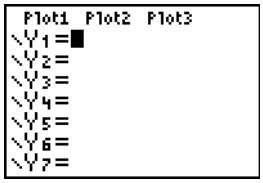
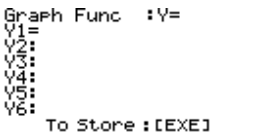
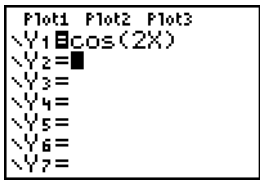
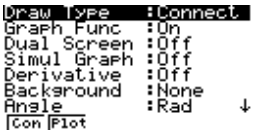
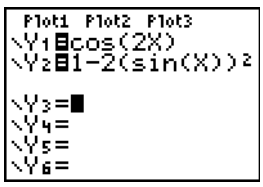
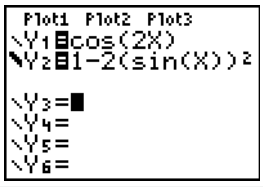
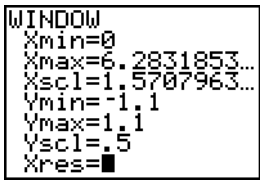
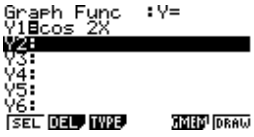
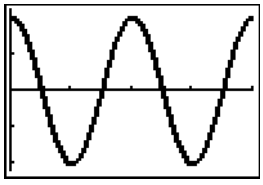
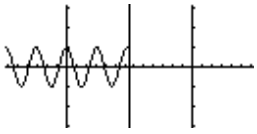
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Repeat the steps from Task A using these coefficients: 2, 6, 12, 4, 12, 12.		Repeat the steps from Task A using these coefficients: 2, 6, 12, 4, 12, 24.	
2.	Result of the simultaneous equation solver.		Result of the simultaneous equation solver.	

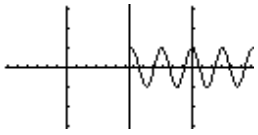
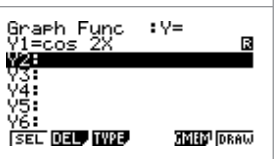
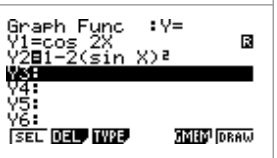
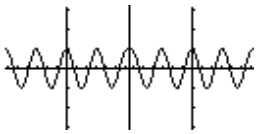
Exploring trigonometric identities

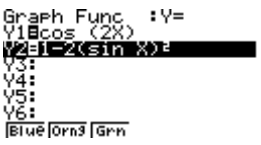
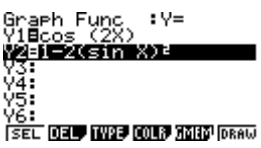
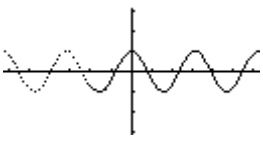
The features available on GDCs vary greatly between models. The TI-84+SE has a feature to change the thickness of the relation graphed. The Casio 9850+ has a colour feature that can be used to compare graphs. Both the Casio FX-9750 Plus and the Casio 9850+ have a dual-graph feature that can be used for this type of exploration.

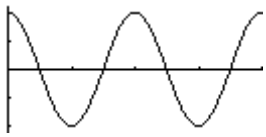
Graphing the relation on either side of a trigonometric identity gives support to the conclusion that the trigonometric identity is true. However, these graphs should not be accepted as adequate demonstrations of the relationship in question, since the fact that the graphs appear to be the same does not guarantee that the identity is true. Including a counter-example in your lesson plan might be a good idea.

Task A: Explore the trigonometric identity $\cos(2\theta) = 1 - 2\sin^2(\theta)$ using the graphical features of a GDC

	TI-84+SE (using graph style)		Casio 9850+ (using dual graph)	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	COS 2 X,T,θ,n) ENTER		SHIFT MENU	
3.	1 − 2 (SIN X,T,θ,n)) x^2 ENTER		DEL ▼ ▼	
4.	◄ ▲ ▼ ENTER to move the cursor to the **left** of Y2. A thick line will then be used for the Y2 graph.		F1 EXE	
5.	Change the window settings to: Xmin = 0 Xmax = 2π Xscl = π/2 Ymin = –1.1 Ymax = 1.1 Yscl = 0.5.			
	WINDOW 0 ENTER 2 2nd ^ ENTER 2nd ^ ÷ 2 ENTER (−) 1 • 1 ENTER 1 • 1 ENTER • 5 ENTER		COS 2 X,θ,T EXE	
6.	GRAPH The graph of $Y1 = \cos 2x$ will appear first. The thicker graph of $Y2 = 1 - 2\sin^2(x)$ will then trace out on top of it.		F6	

7.			OPTN F2	
8.			SHIFT EXIT	
9.			1 − 2 (sin X,θ,T) x^2 EXE	
10.			F6 Note that this image shows two graphs. The one on the left represents $\cos 2\theta$; the one on the right represents $1-2\sin^2\theta$.	

			Casio 9850+ (using colour graphs)	
1.			Enter both functions from the instructions above in the graph screen.	
2.			Change the window settings to: Xmin : -2π max : 2π scale : $\pi/2$ Ymin : −3 max : 3 scale : 1.	
3.			▲ to select and highlight the second graph $1-2\sin^2\theta$.	
4.			F4 F2 EXIT	
5.			F6 The graph of $\cos 2\theta$ will be drawn in dark blue. The graph of $1-2\sin^2\theta$ will draw over it in light orange.	

6.			Change the window settings to: Xmin : 0 max : 2π scale : $\pi/2$ Ymin : –1.6 max : 1.5 scale : 1.	
7.			EXE F6	

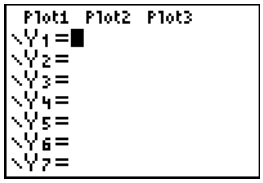
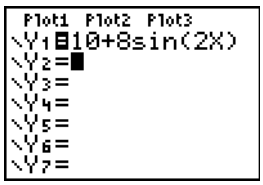
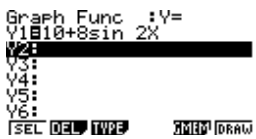
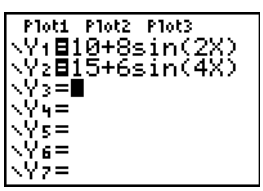
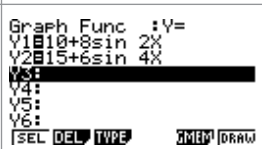
Finding solutions to trigonometric equations

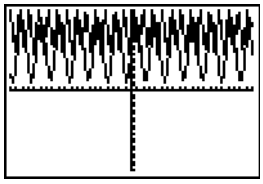
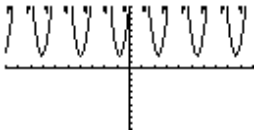
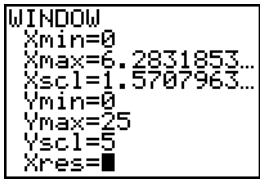
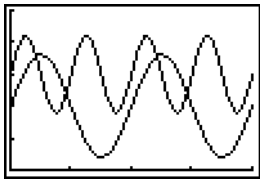
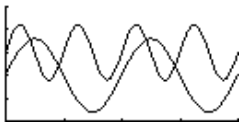
Consider the functions

$f(t) = 10 + 8\sin(2t)$

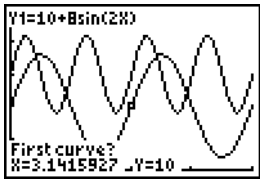
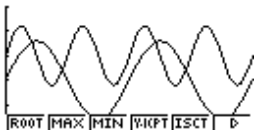
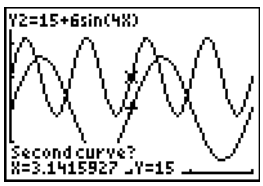
$g(t) = 15 + 6\sin(4t)$

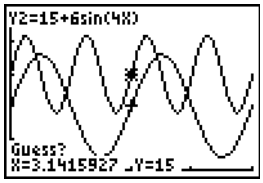
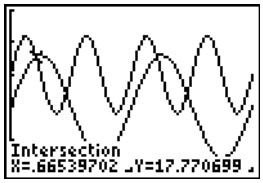
Task A: For how many values of t between 0 and 2π does $f(t) = g(t)$?

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	Change the window settings to: Xmin = –20 Xmax = 20 Xscl = 1 Ymin = –20 Ymax = 20 Yscl = 1.		Change the window settings to: Xmin : –10 max : 10 scale : 1 Ymin : –10 max : 10 scale : 1.	
3.	1 0 + 8 SIN 2 X,T,θ,n) ENTER		1 0 + 8 sin 2 X,θ,T EXE	
4.	1 5 + 6 SIN 4 X,T,θ,n) ENTER		1 5 + 6 sin 4 X,θ,T EXE	

5.	GRAPH		F6	
6.	Change the window settings to: Xmin = 0 Xmax = 2π Xscl = π/2 Ymin = 0 Ymax = 25 Yscl = 5		Change the window settings to: Xmin : 0 max : 2π scale : π/2 Ymin : 0 max : 25 scale : 5	
	WINDOW 0 ENTER 2 2nd ^ ENTER 2nd ^ ÷ 2 ENTER 0 ENTER 2 5 ENTER 5 ENTER		SHIFT F3 0 EXE 2 SHIFT EXP EXE SHIFT EXP ÷ 2 EXE 0 EXE 2 5 EXE 5 EXE	
7.	GRAPH		EXE F6	
8.	Inspect the graphs and observe that there are four points of intersection, which answers Task A.			

Task B: What is the smallest positive value of t for which $f(t) = g(t)$?

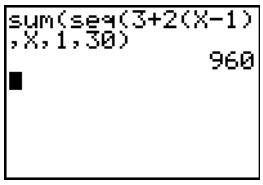
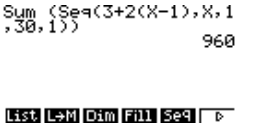
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd TRACE 5		SHIFT	
2.	Make sure the cursor is on the first curve, then ENTER.		F5 F5	

3.	Move the cursor to the first function, then ENTER.		Note: A powerful feature of G-Solve is that the left and right arrow keys will give the other solutions to the left and right of the current one without any further key entry.	
4.	Choose a value as a guess, for example, 0.5. 0 • 5 ENTER			

The solution is given as $x = 0.66539702$.

Summing a sequence

Task A: Find the sum of the first 30 terms of the arithmetic sequence that begins 3, 5, 7, 9...

	TI-84+SE		Casio 9850+	
	The *n*th term is given by: $3 + 2(n-1)$.			
	Press	**Result**	**Press**	**Result**
1.	2nd STAT ◀ 5		MENU 1 OPTN F1 F6 F6 F1	
2.	2nd STAT ◀ 5		(F6 F5	
3.	3 + 2 (X,T,θ,n − 1) , X,T,θ,n , 1 , 3 0) ENTER		3 + 2 (X,θ,T − 1) , X,θ,T , 1 , 3 0 , 1)) EXE	

The answer is 960.

Task B: Find the sum of the first 10 terms of the geometric sequence that begins 320, 240, 180, 135...

	TI-84+SE		Casio 9850+	
	The nth term is given by: $320\ (0.75)^{n-1}$.			
	Press	**Result**	**Press**	**Result**
1.	2nd STAT ◊ 5	sum(	MENU 1 OPTN F1 F6 F6 F1	Sum Sum Prod Cuml ∑ ∫
2.	2nd STAT ◊ 5	sum(seq(	(F6 F5	Sum (Seq(List L→M Dim Fill Seq
3.	3 2 0 (• 7 5) ^ (X,T,θ,n − 1) , X,T,θ,n , 1 , 1 0) ENTER	sum(seq(320(.75) ^(X-1),X,1,10) 1207.918701	3 2 0 (• 7 5) ^ (X,θ,T − 1) , X,θ,T , 1 , 1 0 , 1) EXE	Sum (Seq(320(.75)^(X- 1),X,1,10,1) 1207.918701 List L→M Dim Fill Seq

The answer is 1207.918701.

Functions and calculus

Finding numerical derivatives

One method of approximating the value of the derivative of the function, f, at $x = a$ is to calculate the value of the expression $\frac{f(a+h)-f(a-h)}{2h}$ for small values of h and to examine the limit as h approaches zero. Most GDCs have built-in functions to calculate the value of the expression at different values of h.

Task A: Find the value of the derivative of $f(x) = 2^x$ at $x = 0, 1, 2$ and 3

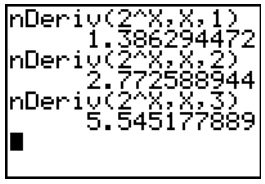
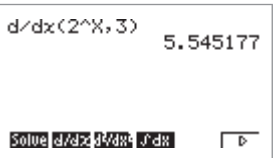
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	MATH 8	nDeriv(	MENU 1 OPTN F4 F2	d/dx(Solve d/dx ∫dx ∫'dx

2.	2 ^ X,T,θ,n , X,T,θ,n , 0 , . 1) ENTER	nDeriv(2^X,X,0,.1) .693702355	2 ^ X,θ,T , 0 , . 1) EXE	d/dx(2^X,0,.1) 0.693702355 Solve d/dx ∫dx ∫'dx
3.	2nd ENTER to paste the previous calculation.	nDeriv(2^X,X,0,.1) .693702355 nDeriv(2^X,X,0,.1)	◀ to paste the previous calculation.	d/dx(2^X,0,.1)
4.	Use the arrow keys to change 0.1 to 0.01. ◀ ◀ 0 1) ENTER	nDeriv(2^X,X,0,.1) .693702355 nDeriv(2^X,X,0,.01) .693152731	Use the arrow keys to change 0.1 to 0.01. ◀ ◀ 0 1) EXE	d/dx(2^X,0,.01) 0.693152731
5.	Use the arrow keys to change 0.01 to 0.001. 2nd ENTER ◀ ◀ 0 1) ENTER	.693702355 nDeriv(2^X,X,0,.01) .693152731 nDeriv(2^X,X,0,.001) .6931472361	Use the arrow keys to change 0.01 to 0.001. ◀ ◀ ◀ 0 1) EXE	d/dx(2^X,0,.001) 0.6931472361
6.	Use the arrow keys to change 0.001 to 0.0001. 2nd ENTER ◀ ◀ 0 1) ENTER	.693152731 nDeriv(2^X,X,0,.001) .6931472361 nDeriv(2^X,X,0,.0001) .6931471812	Use the arrow keys to change 0.001 to 0.0001. ◀ ◀ ◀ 0 1) EXE	d/dx(2^X,0,.0001) 0.6931471811
	If the value of *h* is not specifically entered, the calculator takes the default value of 0.001. The derivatives at 1, 2 and 3 will be approximated with the default value.		The limit appears to be 0.6931471811. This is the value obtained if the value of *h* is omitted from the calculation.	

It can be concluded that the value of the derivative of 2^x at $x = 0$ is 0.693147.

Task B: Compare each value with the value of 2^x at the same value of x

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	MATH 8 2 ^ X,T,θ,n , X,T,θ,n , 1) ENTER	nDeriv(2^X,X,1) 1.386294472	MENU 1 OPTN F4 F2 2 ^ X,θ,T , 1) EXE	d/dx(2^X,1) 1.3862945 Solve d/dx ∫dx ∫'dx
2.	Change the 1 to 2 2nd ENTER ◀ ◀ 2 ENTER	nDeriv(2^X,X,1) 1.386294472 nDeriv(2^X,X,2) 2.772588944	Use the arrow keys to change the 1 to 2. 2 EXE	d/dx(2^X,2) 2.772588 Solve d/dx ∫dx ∫'dx

3.	Change the 2 to 3. 2nd ENTER ◀ ◀ 3 ENTER		Use the arrow keys to change the 2 to 3 3 EXE	

The following table summarizes these results and makes the requested comparisons.

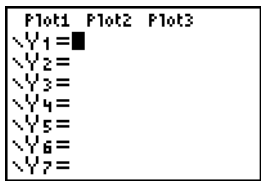
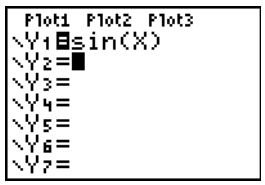
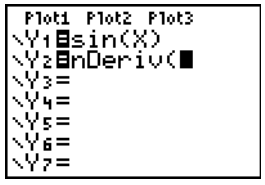
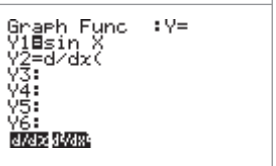
x	2^x	Derivative of 2^x	Derivative of 2^x / 2^x
0	1	0.693147	0.693147
1	2	1.386294	0.693147
2	4	2.772589	0.693147
3	8	5.545178	0.693147

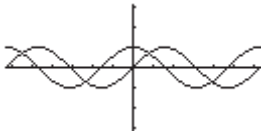
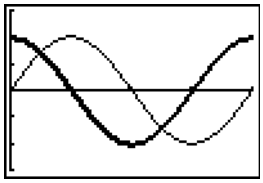
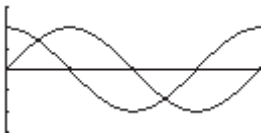
It appears that the derivative of 2^x is equal to 0.693147 times the value of 2^x.

Justifying expressions for derivatives

Sometimes, the necessary mathematics is not available to provide a proper proof for the derivative of a particular function. As with trigonometric identities, the equivalence of expressions can be verified using the graphing features of the calculator together, in this case, with the numerical derivative.

Task A: Show that if $f(x) = \sin x$ and x is in radians, then $f'(x) = \cos x$

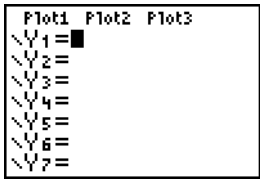
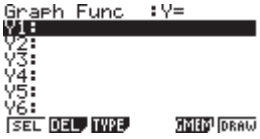
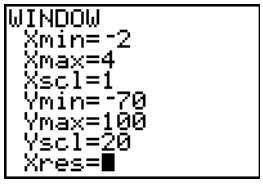
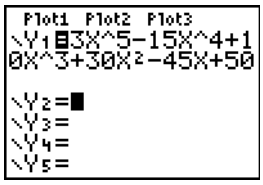
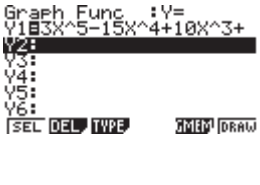
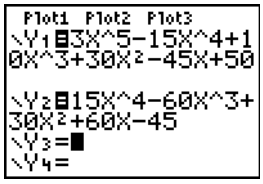
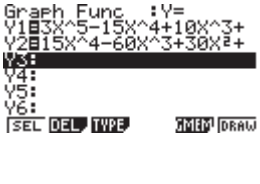
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	SIN X,T,θ,n) ENTER		sin X,θ,T EXE	
3.	MATH 8		OPTN F2 F1	

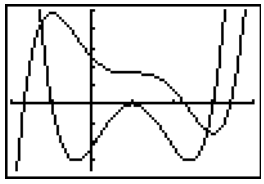
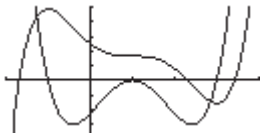
4.	VARS ◀ 1 1 , X,T,θ,n , X,T,θ,n) ENTER	Plot1 Plot2 Plot3 \Y1=sin(X) \Y2=nDeriv(Y1,X,X) \Y3= \Y4= \Y5= \Y6=	sin X,T,θ,T , X,T,θ,T) EXE	Graph Func :Y= Y1=sin X Y2=d/dx(sin X,X) Y3: Y4: Y5: Y6: SEL DEL TYPE GMEM DRAW
5.	cos X,T,θ,n)	Plot1 Plot2 Plot3 \Y1=sin(X) \Y2=nDeriv(Y1,X,X) \Y3=cos(X) \Y4= \Y5= \Y6=	cos X,T,θ,T EXE	Graph Func :Y= Y1=sin X Y2=d/dx(sin X,X) Y3=cos X Y4: Y5: Y6: SEL DEL TYPE GMEM DRAW
6.	◀ ◀ ENTER to move the cursor to the left of Y3. A thick line then will be used for the Y3 graph.	Plot1 Plot2 Plot3 \Y1=sin(X) \Y2=nDeriv(Y1,X,X) \Y3=cos(X) \Y4= \Y5= \Y6=	F6 and then use ▲ and ▼ to select different colours for each function. F1 or blue for Y1. F2 or orange for Y2. F3 or green for Y3.	
7.	Change the window settings to: Xmin = 0 Xmax = 2 x π Xscl = π/2 Ymin = –1.5 Ymax = 1.5 Yscl = 0.5.		Change the window settings to: Xmin : 0 max : 2 x π scale : π/2 Ymin : –1.5 max : 1.5 scale : 0.5.	
	WINDOW 0 ENTER 2 2nd ^ ENTER 2nd ^ ÷ 2 ENTER (–) 1 • 5 ENTER 1 • 5 ENTER • 5 ENTER	WINDOW Xmin=0 Xmax=6.2831853… Xscl=1.5707963… Ymin=-1.5 Ymax=1.5 Yscl=.5 Xres=	SHIFT F3 0 EXE 2 SHIFT EXP EXE SHIFT EXP ÷ 2 EXE (–) 1 • 5 EXE 1 • 5 EXE • 5 EXE	View Window Xmin :0 max :6.2831853 scale:1.57079632 Ymin :-1.5 max :1.5 scale:0.5 INIT TRIG STD STO RCL
8.	GRAPH The graph of cos x should overlay the graph of the derivative of sin x.		EXE F6 The graphs of Y1, Y2 and Y3 are displayed in order. When Y3 is drawn, it clearly overlays Y2.	

Examining graphs of functions and their first and second derivatives

The purpose of this exercise is to look at the graph of a function and its derivative and see how the gradient of the graph of the function is given by the value of the derivative, noting particularly points where the value of the derivative is zero or has a local maximum or minimum.

Task A: **Graph** $f(x) = 3x^5 - 15x^4 + 10x^3 + 30x^2 - 45x + 50$ **and its derivative**
$f'(x) = 15x^4 - 60x^3 + 30x^2 + 60x - 45$

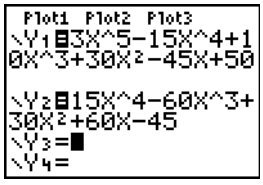
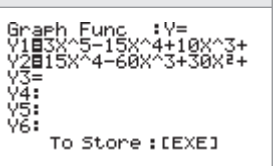
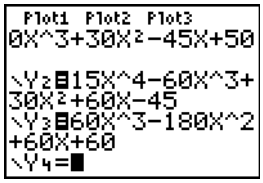
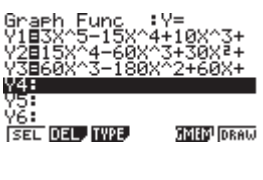
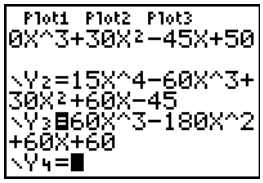
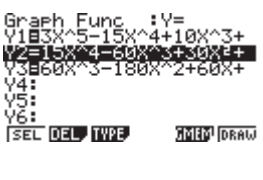
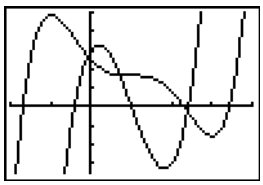
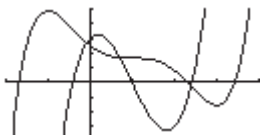
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	Change the window settings to: Xmin = –2 Xmax = 4 Xscl = 1 Ymin = –70 Ymax = 100 Yscl = 20.		Change the window settings to: Xmin : –2 max : 4 scale : 1 Ymin : –70 max : 100 scale : 20.	
	WINDOW (–) 2 ENTER 4 ENTER 1 ENTER (–) 7 0 ENTER 1 0 0 ENTER 2 0 ENTER		SHIFT F3 (–) 2 EXE 4 EXE 1 EXE (–) 7 0 EXE 1 0 0 EXE 2 0 EXE EXE	
3.	Enter $f(x)$ into Y1. 3 X,T,θ,n ^ 5 − 1 5 X,T,θ,n ^ 4 + 1 0 X,T,θ,n ^ 3 + 3 0 X,T,θ,n x^2 − 4 5 X,T,θ,n + 5 0 ENTER		Enter $f(x)$ into Y1. 3 X,θ,T ^ 5 − 1 5 X,θ,T ^ 4 + 1 0 X,θ,T ^ 3 + 3 0 X,θ,T x^2 − 4 5 X,θ,T + 5 0 EXE	
4.	Enter $f'(x)$ into Y2. 1 5 X,T,θ,n ^ 4 − 6 0 X,T,θ,n ^ 3 + 3 0 X,T,θ,n x^2 + 6 0 X,T,θ,n − 4 5 ENTER		Enter $f'(x)$ into Y2. 1 5 X,θ,T ^ 4 − 6 0 X,θ,T ^ 3 + 3 0 X,θ,T x^2 + 6 0 X,θ,T − 4 5 EXE	

5.	GRAPH		F6	

It should be noted that wherever the gradient is negative the derivative is negative, and wherever the gradient is positive the derivative is positive, the derivative having its most extreme values when the gradient is most positive or most negative. It should also be noted that wherever the derivative is zero, the tangent to the graph of f is horizontal and these situations correspond to a local maximum, a local minimum and a horizontal (stationary) point of inflexion. Points of inflexion should also be seen to occur where the derivative has local maximum and local minimum values.

Task B: Graph $f(x) = 3x^5 - 15x^4 + 10x^3 + 30x^2 - 45x + 50$ and its second derivative $f''(x) = 60x^3 - 180x^2 + 60x + 60$

The purpose of this next exercise is to look at the graph of a function and its second derivative, and see how the concavity of the graph is related to whether the second derivative is positive or negative, also noting where the second derivative is zero.

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Use the arrow keys to scroll down to Y3.		SHIFT EXIT Use the arrow keys to scroll down to Y3.	
2.	Enter $f''(x)$ into Y3. 6 0 X,T,θ,n ^ 3 − 1 8 0 X,T,θ,n ^ 2 + 6 0 X,T,θ,n + 6 0 ENTER		Enter $f''(x)$ into Y3. 6 0 X,θ,T ^ 3 − 1 8 0 X,θ,T ^ 2 + 6 0 X,θ,T + 6 0 EXE	
3.	To stop Y2 from displaying, use the arrow keys to move the cursor over the = sign on Y2 and then ENTER. The black square over the = sign will disappear.		To stop Y2 from displaying, use the arrow keys to move the cursor over the = sign on Y2 and then F1. The black square over the = sign will disappear.	
4.	GRAPH		F6	

It should be noted that wherever the second derivative is positive, the graph opens upwards (concave up) and wherever the second derivative is negative, the graph opens downwards (concave down). The relationship between where the second derivative is zero and points of inflexion should be seen.

Simultaneous display of all three graphs can give rise to further analysis. The point should be made that $f'' = 0$ does not necessarily mean a point of inflexion (for example, $y = x^4$).

The "Colour Graph" option on the Casio 9850+, or the "Graph Styles" option on the TI-84+SE, are useful functionalities to compare the graphs in these examples.

Using the numerical integration facility

The numerical integration facility of most GDCs can be used to provide a good approximation of the values of integrals that either could not be obtained by traditional means, or would take lengthy and/or complex calculation. It can also be used to check values of integrals obtained by traditional means. The following example is taken from the May 2003 mathematical methods SL paper 2 examination.

Task A: Evaluate $\int_0^{\frac{3\pi}{4}} e^x(\sin x + \cos x)\,dx$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	MATH 9	fnInt(▮	MENU 1 OPTN F4 F4	∫(Solve d/dx ∫dx ∫dx
2.	2nd LN X,T,θ,n) (SIN X,T,θ,n) + COS X,T,θ,n)) , X,T,θ,n , 0 , 3 2nd ^ ÷ 4) ENTER	fnInt(e^(X)(sin(X)+cos(X)),X,0,3π/4) 7.460488539 ▮	SHIFT ln X,θ,T (sin X,θ,T + COS X,θ,T) , 0 , 3 SHIFT EXP ÷ 4) EXE	∫(eX(sin X+cos X),0,3π÷4) 7.460488539 Solve d/dx ∫dx ∫dx

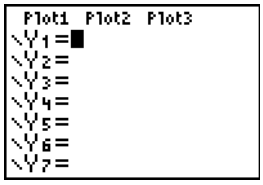
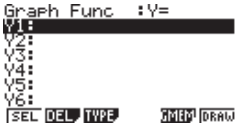
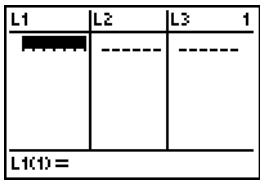
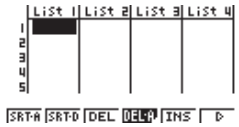
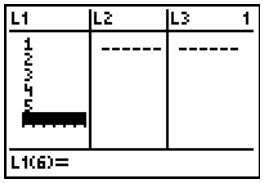
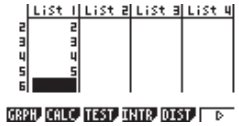
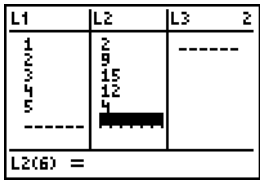
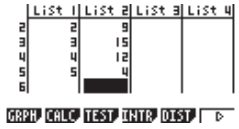
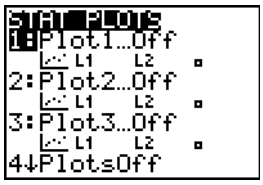
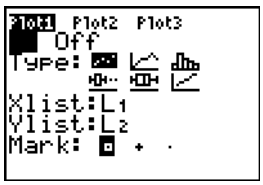
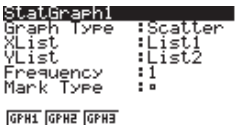
The value of the integral is 7.46.

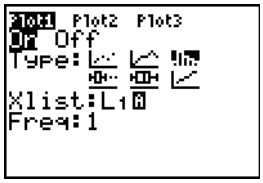
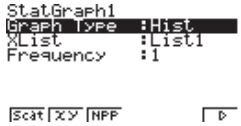
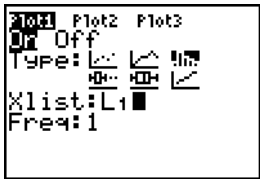
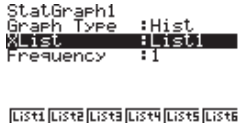
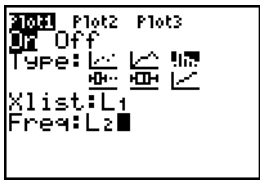
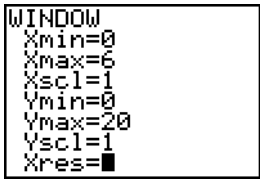
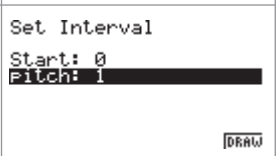
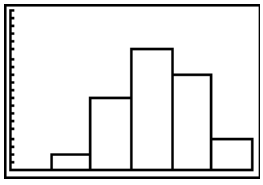
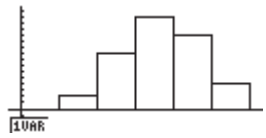
Probability and statistics

Drawing histograms using lists

Task A: Draw a histogram for the following data

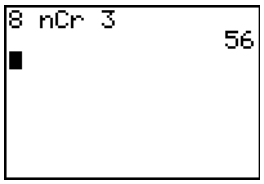
x	1	2	3	4	5
f	2	9	15	12	4

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	STAT 1		MENU 2 Clear any data currently in lists. ▲ F6 F4 F1 for each list that contains data.	
3.	Enter data into List 1. 1 ENTER 2 ENTER 3 ENTER 4 ENTER 5 ENTER		Enter data into List 1. 1 EXE 2 EXE 3 EXE 4 EXE 5 EXE	
4.	Enter data into List 2. ▶ 2 ENTER 9 ENTER 1 5 ENTER 1 2 ENTER 4 ENTER		Enter data into List 2. ▶ 2 EXE 9 EXE 1 5 EXE 1 2 EXE 4 EXE	
5.	2nd Y=			
6.	ENTER ENTER to select "On".		F1 F6	

7.	◀ ▶ ▶ ENTER to select the bar graph icon.		▼ F6 F1 to ensure that Graph Type is set to Histogram.	
8.	2nd STAT 1 to insert L_1 into the Xlist line.		▼ F1 to ensure that XList is set to List 1.	
9.	2nd STAT 2 to insert L_2 into the Freq line.		▼ F3 to ensure that Frequency is set to List 2.	
10.	Change the window settings to: Xmin = 0 Xmax = 6 Xscl = 1 Ymin = 0 Ymax = 20 Yscl = 1.		Change the window settings to: Xmin : 0 max : 6 scale : 1 Ymin : 0 max : 20 scale : 1.	
	WINDOW 0 ENTER 6 ENTER 1 ENTER 0 ENTER 2 0 ENTER 1 ENTER		EXIT SHIFT F3 0 EXE 6 EXE 1 EXE 0 EXE 2 0 EXE 1 EXE	
11.	Please note: In L_1 the frequency for one is plotted from one to two, so the x-axis needs to be one more than five.		EXIT F1 F1 Set Start to 0 and Pitch to 1. 0 EXE 1 EXE	
12.	GRAPH		F6	

Evaluating $\binom{n}{r}$

Task A: Find $\binom{8}{3}$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	8 MATH ◀ 3		MENU 1 OPTN F6 F3	
2.	3 ENTER		8 F3 3 EXE	

The answer is $\binom{8}{3} = 56$.

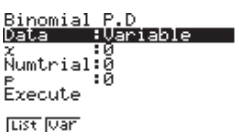
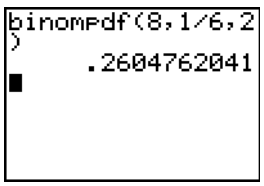
Finding binomial probabilities

Example: Eight fair six-sided dice are rolled.

Task A: Find the probability of obtaining exactly two sixes

This situation represents a binomial probability situation with $n = 8,\ p = \frac{1}{6}$.

We want: $\mathrm{P}(x = 2)$.

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd VARS 0		MENU 2 F5 F5 F1 F2	
2.	8 , 1 ÷ 6 , 2) ENTER		▼ 2 EXE 8 EXE 1 ÷ 6 EXE EXE	

The answer is 0.2605.

Task B: Find the probability of obtaining at most two sixes

We want: $\mathrm{P}(x \leq 2)$.

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	2nd VARS ALPHA MATH	binompdf(8,1/6,2) .2604762041 binomcdf(	EXIT EXIT F5 F5 F2 F2	Binomial C.D Data :Variable x :0 Numtrial:0 p :0 Execute List Var
2.	8 , 1 ÷ 6 , 2) ENTER	binompdf(8,1/6,2) .2604762041 binomcdf(8,1/6,2) .8651531068	▼ 2 EXE 8 EXE 1 ÷ 6 EXE EXE	Binomial C.D P(x)=0.86515

The answer is 0.8652.

Task C: Find the probability of obtaining at least two sixes

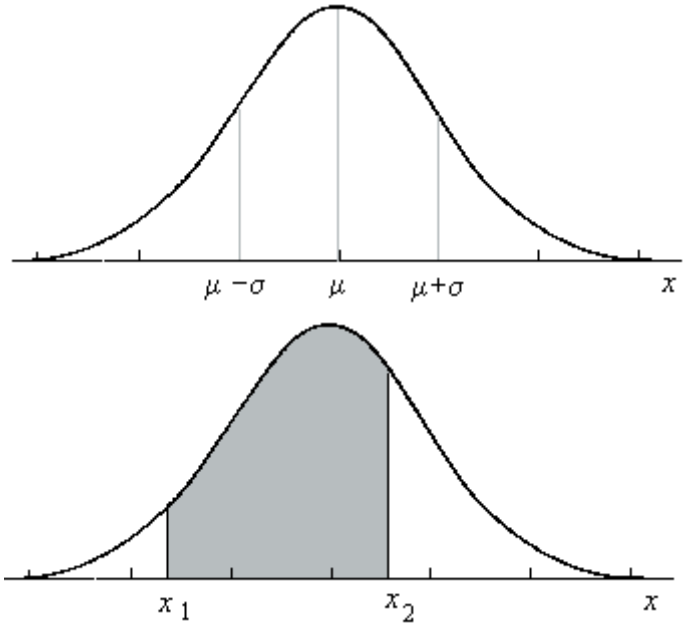
We want: $\mathrm{P}(x \geq 2)$.

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
	The answer will be $1-\left[p(0)+p(1)\right]$.			
1.	1 − 2nd VARS ALPHA MATH	binompdf(8,1/6,2) .2604762041 binomcdf(8,1/6,2) .8651531068 1-binomcdf(	EXIT EXIT F5 F5 F2 F2	Binomial C.D Data :Variable x :2 Numtrial:8 p :0.16666666 Execute List Var
2.	8 , 1 ÷ 6 , 1) ENTER	.2604762041 binomcdf(8,1/6,2) .8651531068 1-binomcdf(8,1/6 ,1) .3953230977	▼ 1 EXE 8 EXE 1 ÷ 6 EXE EXE	Binomial C.D P(x)=0.60467
3.			MENU 1 1 − SHIFT (−) EXE	1-Ans 0.3953230977

The answer is 0.3953.

Finding normal probabilities

The normal probability density function, $\phi(x)$, describes the probability distribution of the continuous random variable, X, in terms of the mean of its distribution, μ, and its standard deviation, σ. Many common random variables have this distribution. The graph of $\phi(x)$ has the characteristic bell shape. It is symmetric about the mean, μ, and the points of inflexion on the bell curve occur at $\mu \pm \sigma$.



The total area under the curve is equal to one, and the probability that the value of X will lie between the values x_1 and x_2 is equal to the area under the curve between these two values. Although it is not necessary to know the exact form of the function for basic probability calculations, it is given by $\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$. The standard normal probability density function for a variable z with mean of zero and standard deviation of one is given by $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$.

The general normal distribution can be transformed into the standard normal distribution by the transformation $z = \frac{x-\mu}{\sigma}$. With traditional normal tables, and with some calculators, it is necessary to make this transformation. With other calculators, this transformation can often be avoided.

Consider the following typical problem involving a normally distributed variable: a normally distributed variable, X, has a mean equal to 100 and a standard deviation equal to 15.

Task A: Find the probability that $90 < X < 120$

	TI-84+SE		Casio 9850+	
	Use the normal cumulative distribution function (normalcdf).			
	Press	**Result**	**Press**	**Result**
1.	2nd VARS 2	normalcdf(	MENU 2 F5 F1 F2	Normal C.D Lower :0 Upper :0 σ :0 μ :0 Execute

2.	Enter four parameters: (lower limit, upper limit, mean, standard deviation). 9 0 , 1 2 0 , 1 0 0 , 1 5) ENTER	normalcdf(90,120 ,100,15) .6562962511	Enter four parameters: (lower limit, upper limit, standard deviation, mean). 9 0 EXE 1 2 0 EXE 1 5 EXE 1 0 0 EXE EXE	Normal C.D Prob=0.65629

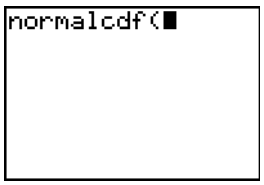
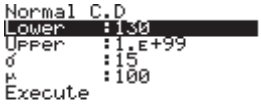
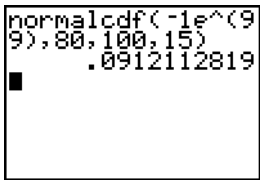
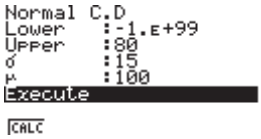
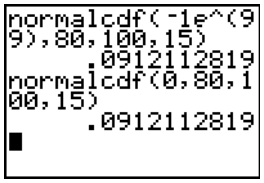
The answer is 0.656.

Task B: Find the probability that $X > 130$

Theoretically, this requires an upper limit of $+\infty$, and part (c) will require a lower limit of $-\infty$. The values $1e^{99}$ and $-1e^{99}$ respectively may be used, but more convenient values (at least five standard deviations from the mean) may be used to achieve answers to the same accuracy.

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd VARS 2	normalcdf(▮	EXE to return to Normal C.D screen.	Normal C.D Lower :90 Upper :120 σ :15 μ :100 Execute
2.	1 3 0 , 1 2nd LN 9 9) , 1 0 0 , 1 5) ENTER	normalcdf(130,1e ^(99),100,15) .022750062	1 3 0 EXE 1 EXP 9 9 EXE 1 5 EXE 1 0 0 EXE	Normal C.D Lower :130 Upper :1.E+99 σ :15 μ :100 Execute CALC
3.	This gives the answer 0.0228 to three significant figures. Changing the $1e^{99}$ to 1,000 gives an answer identical to this one to at least 10 decimal places. 2nd VARS 2 1 3 0 , 1 0 0 0 , 1 0 0 , 1 5) ENTER	normalcdf(130,1e ^(99),100,15) .022750062 normalcdf(130,10 00,100,15) .022750062	EXE	Normal C.D Prob=0.02275

Task C: Find the probability that $X < 80$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd VARS 2		EXIT to return to Normal C.D screen.	
2.	(−) 1 2nd LN 9 9) , 8 0 , 1 0 0 , 1 5) ENTER		(−) 1 EXP 9 9 EXE 8 0 EXE 1 5 EXE 1 0 0 EXE	
3.	This gives the answer 0.0912 to three significant figures. Changing the $-1e^{99}$ to zero gives an answer identical to this one to at least 10 decimal places. 2nd ENTER to display previous entry and use the arrow keys and DEL to edit.		EXE	
	Note that as the last two parameters (the mean and standard deviation) have not been entered, the calculator assumes a mean of zero and a standard deviation of one, corresponding to the standardized normal distribution. If the calculator is used in this way, the lower and upper limits must first be converted to standardized values.			

Finding inverse normal probabilities

A typical problem in which the probabilities for a normally distributed variable are known and the values of the variable corresponding to these probabilities are desired could be the following: a normally distributed variable, X, has mean 500 and standard deviation 100.

Task A: Find the value a, if 5% of the values of X are less than a

Again, depending on the model of GDC being used, it may or may not be necessary to solve this problem using the standardized normal distribution.

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	In all cases, begin by accessing the inverse normal function through the sequence 2nd VARS 3.	invNorm(	MENU 2 F5 F1 F3	Inverse Normal Area :0 σ :0 μ :0 Execute
2.	The parameters for invNorm are (probability less than, mean, standard deviation).			
	• 0 5 , 5 0 0 , 1 0 0) ENTER	invNorm(.05,500,100) 335.5146374	• 0 5 EXE 1 0 0 EXE 5 0 0 EXE EXE	Inverse Normal x=335.51

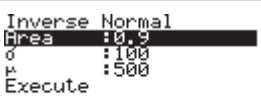
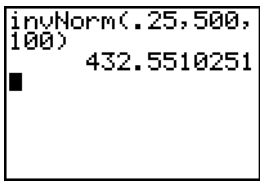
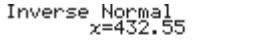
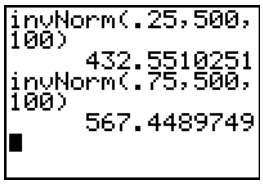
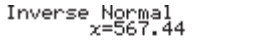
The answer is $a = 335.5$.

Task B: Find the value b, if 10% of the values of X are greater than b

	TI-84+SE		Casio 9850+	
	If the probability of greater than *b* is 0.10, the probability of less than *b* is 0.90.			
	Press	**Result**	**Press**	Result
1.	In all cases, begin by accessing the inverse normal function through the sequence 2nd VARS 3.	invNorm(	EXE	Inverse Normal Area :0.05 σ :100 μ :500 Execute
2.	• 9 , 5 0 0 , 1 0 0) ENTER	invNorm(.9,500,100) 628.1551567	• 9 EXE 1 0 0 EXE 5 0 0 EXE EXE	Inverse Normal x=628.15

The answer is $b = 628.2$.

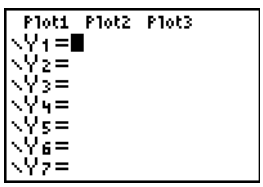
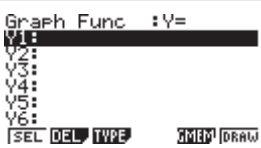
Task C: The middle 50% of the values of X lie between c and d. Find c and d

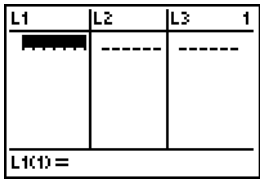
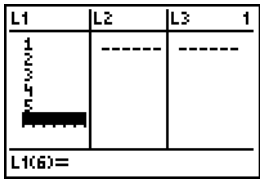
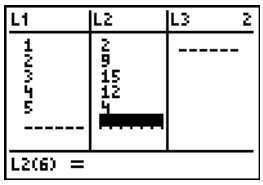
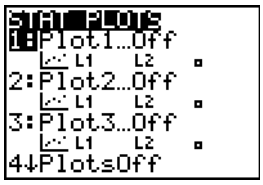
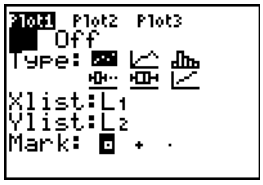
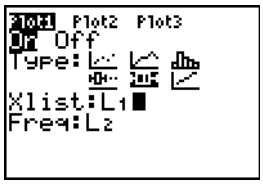
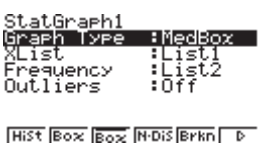
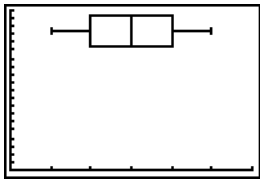
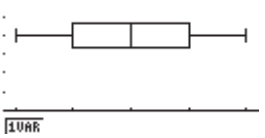
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	The probability of less than *c* must be 0.25 and less than *d* must be 0.75. 2nd VARS 3		EXIT Boundaries for area will be values separating the bottom 25% and the bottom 75%.	
2.	To find c: • 2 5 , 5 0 0 , 1 0 0) ENTER.		To find c: • 2 5 EXE EXE.	
	$c = 432.6$			
3.	Change the 0.25 to 0.75. 2nd ENTER ◀ ▶ ▶ 7 ENTER		Change the 0.25 to 0.75. EXIT • 7 5 EXE EXE	
	The answer is $d = 567.4$. So the middle 50% of values lie between 432.6 and 567.4.			

Drawing box and whisker plots

Task A: Draw a box and whisker plot to represent the following data

x	1	2	3	4	5
f	2	9	15	12	4

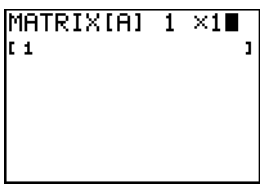
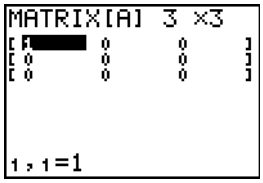
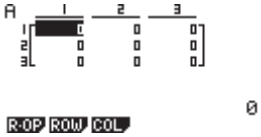
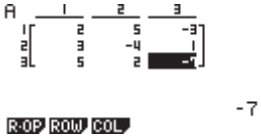
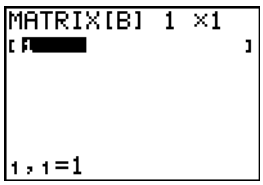
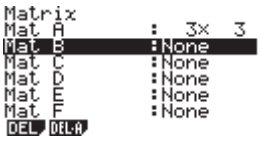
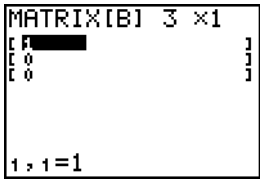
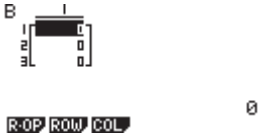
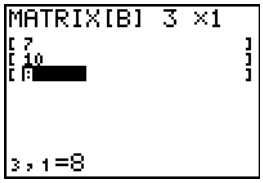
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs.		MENU 5 Remove any stored graphs.	
2.	Change the window settings to: Xmin = 0 Xmax = 5 Xscl = 1 Ymin = 0 Ymax = 10 Yscl = 1.		Change the window settings to: Xmin : 0 max : 5 scale : 1 Ymin : 0 max : 10 scale : 1.	

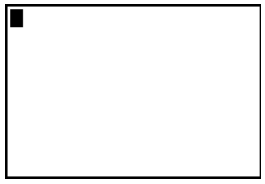
3.	STAT 1 Note that if the exercise on drawing histograms has already been done, the values will already have been entered. If this is the case, go straight to step 5.		MENU 2 Clear any data currently in lists. ▲ F6 F4 F1 for each list that contains data.	
4.	Enter data into List 1. 1 ENTER 2 ENTER 3 ENTER 4 ENTER 5 ENTER		Enter data into List 1. 1 EXE 2 EXE 3 EXE 4 EXE 5 EXE	
5.	Enter data into List 2. ▶ 2 ENTER 9 ENTER 1 5 ENTER 1 2 ENTER 4 ENTER		Enter data into List 2. ▶ 2 EXE 9 EXE 1 5 EXE 1 2 EXE 4 EXE	
6.	2nd Y=			
7.	ENTER ENTER to select "On".		MENU 2 F1 F6	
8.	◀ ▶ ▶ ▶ ▶ ENTER		▼ F6 F2	
9.	GRAPH		EXIT F1	

Matrices and vectors

Entering matrices into the calculator

Task A: Enter the following two matrices: $A = \begin{pmatrix} 2 & 5 & -3 \\ 3 & -4 & 1 \\ 5 & 2 & -7 \end{pmatrix}$, $B = \begin{pmatrix} 7 \\ 10 \\ 8 \end{pmatrix}$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd x^{-1} ◆ 1		MENU 3	
2.	3 ENTER 3 ENTER		3 EXE 3 EXE	
3.	Enter the values for matrix A. 2 ENTER 5 ENTER (−) 3 ENTER 3 ENTER (−) 4 ENTER 1 ENTER 5 ENTER 2 ENTER (−) 7 ENTER		Enter the values for matrix A. 2 EXE 5 EXE (−) 3 EXE 3 EXE (−) 4 EXE 1 EXE 5 EXE 2 EXE (−) 7 EXE	
4.	2nd x^{-1} ◆ 2		EXIT ▼	
5.	3 ENTER 1 ENTER		3 EXE 1 EXE	
6.	7 ENTER 1 0 ENTER 8 ENTER		7 EXE 1 0 EXE 8 EXE	

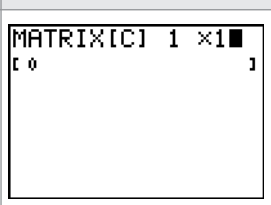
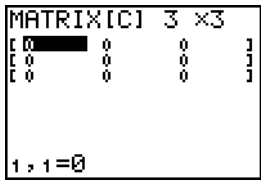
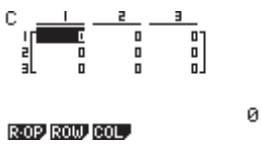
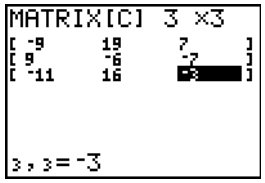
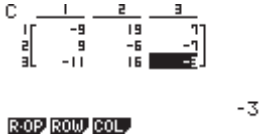
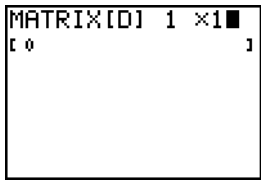
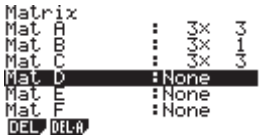
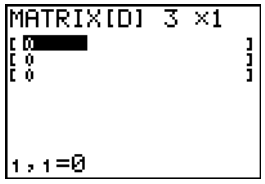
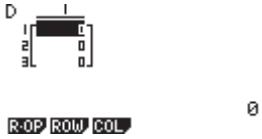
7.	If needed, 2nd MODE CLEAR MATH to get back to the home screen.		If needed, MENU 1 to get back to the home screen.	

Matrix algebra

Enter the following matrices, using the procedures outlined in the "Entering matrices into the calculator" section.

$$A = \begin{pmatrix} 2 & 5 & -3 \\ 3 & -4 & 1 \\ 5 & 2 & -7 \end{pmatrix}, \quad B = \begin{pmatrix} 7 \\ 10 \\ 8 \end{pmatrix}, \quad C = \begin{pmatrix} -9 & 19 & 7 \\ 9 & -6 & -7 \\ -11 & 16 & -3 \end{pmatrix} \text{ and } D = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}.$$

Task A: Enter the matrices C and D into the calculator. (A and B should already be in the memory from the above exercise. If not, enter them again now.)

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd x^{-1} ◀ 3		MENU 3 ▼ ▼	
2.	3 ENTER 3 ENTER		3 EXE 3 EXE	
3.	(−) 9 ENTER 1 9 ENTER 7 ENTER 9 ENTER (−) 6 ENTER (−) 7 ENTER (−) 1 1 ENTER 1 6 ENTER (−) 3 ENTER		(−) 9 EXE 1 9 EXE 7 EXE 9 EXE (−) 6 EXE (−) 7 EXE (−) 1 1 EXE 1 6 EXE (−) 3 EXE	
4.	2nd x^{-1} ◀ 4		EXIT ▼	
5.	3 ENTER 1 ENTER		3 EXE 1 EXE	

6.	8 ENTER 5 ENTER (−) 9 ENTER	MATRIX[D] 3 ×1 [8] [5] [-9] 3,1=-9	8 EXE 5 EXE (−) 9 EXE	D I 1 8 2 5 3 -9 -9 R▸OP ROW COL

Task B: Calculate $3\boldsymbol{B} - 2\boldsymbol{D}$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	3 2nd x^{-1} 2 − 2 2nd x^{-1} 4	3[B]-2[D] [ ]	MENU 1 OPTN F2 A new list of options appears above the Function keys. The matrix calculations can now be done.	Mat Mat▸L Det Trn Aug ▷
2.	ENTER	3[B]-2[D] [[5] [20] [42]] [ ]	3 F1 ALPHA log − 2 F1 ALPHA sin EXE	Ans I 1 5 2 20 3 42 5 Mat Mat▸L Det Trn Aug ▷

The answer is $3\boldsymbol{B} - 2\boldsymbol{D} = \begin{pmatrix} 5 \\ 20 \\ 42 \end{pmatrix}$.

Task C: Calculate $5\boldsymbol{A} + \boldsymbol{C}$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	5 2nd x^{-1} 1 + 2nd x^{-1} 3	3[B]-2[D] [[5] [20] [42]] 5[A]+[C] [ ]	5 F1 ALPHA X,θ,T + F1 ALPHA ln	3Mat B−2Mat D 5Mat A+Mat C Done Mat Mat▸L Det Trn Aug ▷
2.	ENTER	[[5] [20] [42]] 5[A]+[C] [[1 44 -8] [24 -26 -2] [14 26 -38]] [ ]	EXE	Ans 1 2 3 1 1 44 -8 2 24 -26 -2 3 14 26 -38 1 Mat Mat▸L Det Trn Aug ▷

The answer is $5\boldsymbol{A} + \boldsymbol{C} = \begin{pmatrix} 1 & 44 & -8 \\ 24 & -26 & -2 \\ 14 & 26 & -38 \end{pmatrix}$.

Task D: Calculate the product of A and B

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	2nd x^{-1} 1 2nd x^{-1} 2	[[5] [20] [42]] 5[A]+[C] [[1 44 -8] [24 -26 -2] [14 26 -38]] [A][B]	ALPHA X,θ,T × ALPHA log	3Mat B−2Mat D Done 5Mat A+Mat C Done A×B Mat Mat⇒L Det Trn Aug
2.	ENTER	[[1 44 -8] [24 -26 -2] [14 26 -38]] [A][B] [[40] [-11] [-1]]	EXE	3Mat B−2Mat D Done 5Mat A+Mat C Done A×B 0 Mat Mat⇒L Det Trn Aug

The answer is $\boldsymbol{AB} = \begin{pmatrix} 40 \\ -11 \\ -1 \end{pmatrix}$.

Finding the determinant and inverses of matrices

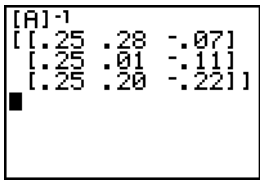
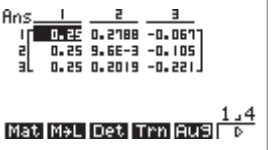
Let $\boldsymbol{A} = \begin{pmatrix} 2 & 5 & -3 \\ 3 & -4 & 1 \\ 5 & 2 & -7 \end{pmatrix}$

Task A: Find the determinant of matrix A

	TI-84+SE		Casio 9850+	
	Press	Result	Press	Result
1.	Enter the matrices A and B as outlined in the "Entering matrices into the calculator" section.			
2.	2nd x^{-1} (ENTER 2nd x^{-1} 1) ENTER	det([A]) 104	MENU 1 OPTN F2 F3 F1 ALPHA X,θ,T EXE	Det Mat A 104 Mat Mat⇒L Det Trn Aug

The determinant of matrix A is 104.

Task B: Find the inverse of matrix A

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	2nd x^{-1} 1 x^{-1} ENTER		MENU 1 OPTN F2 F1 ALPHA F↔D SHIFT) EXE	
2.	The image shown here is from a GDC set to display numbers to two decimal places. (This is done by pressing MODE then setting the float to two by pressing ◀ ▶ ▶ ▶ ENTER.)			

The inverse of matrix A is $\begin{pmatrix} 0.25 & 0.28 & -0.07 \\ 0.25 & 0.01 & -0.11 \\ 0.25 & 0.20 & -0.22 \end{pmatrix}$.

Assessment and the GDC

What students should write down in examinations

The current group 5 mathematics objectives state that students should "organize and present information and data in tabular, graphical and/or diagrammatic forms", and "formulate a mathematical argument and communicate it clearly". This means that it is important for students to learn to communicate effectively in examinations.

The assessment model has changed from previous mathematics courses, and students are now expected to show their working on all papers to achieve full marks. For mathematics HL and SL paper 1, it is no longer the case that full marks will be awarded for providing the correct answer only. To receive full marks on any question, the correct answer will generally need to be supported by suitable working.

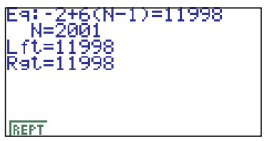
The discussions about what students should write down in examinations have been going on for a long time, even before the advent of the GDC. The answers to the questions "What should be written down in an examination when I have used a calculator?" and "How do I show my working?" also apply to situations when a calculator is not used. The important factor is good communication.

Example question 1

In an arithmetic sequence, the first term is –2, the fourth term is 16, and the nth term is 11,998.

(a) Find the common difference d.

(b) Find the value of n. **[6 marks]**

Write down	Rationale
$u_1 = -2, u_4 = 16, u_n = 11998$ $u_n = u_1 + (n-1)d$	Write down the given information in mathematical language and write down any relevant formula.
(a) $16 = -2 + 3d$ (This gives $d = 6$) (b) $11998 = -2 + (n-1)6$	Set up the equations.
	Use an equation solver (this example uses the Casio 9850+) to compute. 
(a) $d = 6$ (b) $n = 2001$	Write down the answer. Check that the answer matches the question asked and contains no errors.

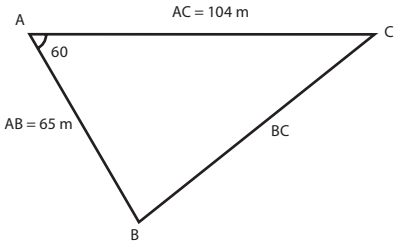
The first step in answering most questions is to extract the information, select an appropriate strategy, and then use the information. Quite often, this involves rearranging the information into a form that is suitable for use. When using a GDC, the information may need to be written in a form suitable for input into the GDC—what some teachers call "getting the question calculator ready". For example, if asked to find the area between curves, students may need to identify the points of intersection and make a note of these. They should also communicate that they are using a definite integral between appropriate limits to find the area. The first part of a question may ask them to write down an integral representing the area but, even if it does not, this should be one of the first steps. Therefore, in the first part of the answer students should explain in mathematical language (not calculator notation) what they are doing.

Students need to give enough information so that the important steps in the solution are apparent. However, it is not necessary for them to write down every single algebraic or arithmetic step. Copying all results from the calculator onto paper would interrupt a chain of thought, be time-consuming and probably increase the likelihood of errors occurring. The challenge is to determine what constitutes an appropriate solution. Teachers should encourage students to identify "key features" of solutions, and make sure they write these down.

Over the last few years, discussions between examiners and teachers attending meetings at the International Baccalaureate Curriculum and Assessment Centre (IBCA) have highlighted that there are many different approaches possible. There are innovative and interesting ways of using a GDC to answer questions that most people would have thought could not be done on a GDC. Various people have been asked to share their thoughts, and some of these appear in this document.

Example question 2

A farmer owns a triangular field ABC. The side AC is 104m, the side AB is 65m and the angle between these two sides is 60°. Calculate the length of the third side of the field.

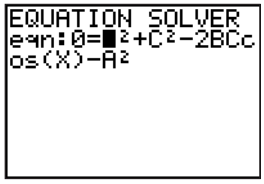
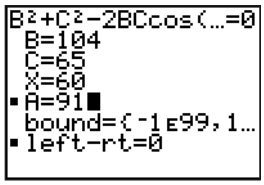
Write down	Rationale
	Draw an appropriate diagram.
$a^2 = b^2 + c^2 - 2bc\cos A$	Identify and write down the appropriate rule to be used (cosine rule).

Two likely methods are possible.

Method 1

Write down	Rationale
$\mathrm{BC}^2 = 65^2 + 104^2 - 2(65)(104)\cos 60^\circ$	Substitute in rule.
$\mathrm{BC}^2 = 8281$	Compute.
$\mathrm{BC} = 91\mathrm{m}$	Write down the answer.

Method 2

Write down	Rationale
$0 = b^2 + c^2 - 2bc\cos A - a^2$	Rewrite the rule equal to zero and enter into the equation solver.  (TI-84+SE)
$\mathrm{BC} = 91\mathrm{m}$	Highlight A and solve.  Write down the answer.

Note that there is generally no "right" or "wrong" use of the GDC. Some questions are written so that they can only be answered using a GDC; some can be answered with or without a GDC, and others are meant to be done analytically.

In particular, it is not appropriate to use a GDC for a question that asks for an exact answer or uses the command term "show that".

"Show that" and answer "carried forward" = find questions

The style of setting several parts to a question often requires students to "carry forward" an answer from one part of the question to another. This "carry forward" plays an important role in the remainder of a question; the answer is often provided and the command term used is "show that".

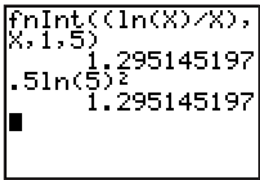
In this situation, teachers should advise students to treat it as a "find" question even though the answer is given. It is a good idea for students to use the given answer as a check that they have written down the question correctly. Even if they fail to do the "show that" part, they should still use the given answer in subsequent parts of a question. The examples below demonstrate what students should write down when answering this type of question and when it may be appropriate for them to use a GDC.

Example question 1

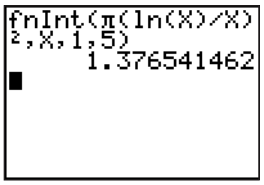
The function f is defined on the domain $x \geq 1$ by $f(x) = \dfrac{\ln x}{x}$. Let R be the region enclosed by the graph of f, the x-axis and the line $x = 5$.

(a) Find the **exact** value of the area of R.

(b) The region R is rotated through an angle of 2π about the x-axis. Find the volume of the solid of revolution generated.

Write down	Rationale
(a) Area = $\int_1^5 \frac{\ln x}{x}\mathrm{d}x$	Write down an appropriate mathematical formula representing the area.
Using the GDC to compute the integral would be inappropriate in this part since the question demands the exact value. An appropriate use of the GDC might be to check the answer.	
$u = \ln x, \mathrm{d}u = \frac{1}{x}\mathrm{d}x$ $\int u\mathrm{d}u = \frac{u^2}{2}\left(= \frac{(\ln x)^2}{2}\right)$ Area = $\left[\frac{(\ln x)^2}{2}\right]_1^5 = \frac{1}{2}\left((\ln 5)^2 - (\ln 1)^2\right)$ Area = $\frac{1}{2}(\ln 5)^2$	Either Find the integral by substitution/ inspection.
$u = \ln x, \mathrm{d}v = \frac{1}{x} \Rightarrow \mathrm{d}u = \frac{1}{x}, v = \ln x$ $I = uv - \int u\mathrm{d}v = (\ln x)^2 - \int \ln x \frac{1}{x}\mathrm{d}x = (\ln x)^2 - I$ $\Rightarrow 2I = (\ln x)^2 \Rightarrow I = \frac{(\ln x)^2}{2}$ $\Rightarrow \text{area} = \left[\frac{(\ln x)^2}{2}\right]_1^5 = \frac{1}{2}\left((\ln 5)^2 - (\ln 1)^2\right)$ Area = $\frac{1}{2}(\ln 5)^2$	Or Find the integral I by parts.
	The appropriate use of the GDC in this example might be to check the answer. 

In part (b), the GDC is appropriate as a computational tool for the definite integration.

Write down	Rationale
(b) $V = \int_a^b \pi y^2 \, \mathrm{d}x$	Write down an appropriate mathematical formula representing the volume.
$= \int_1^5 \pi \left(\frac{\ln x}{x} \right)^2 \mathrm{d}x$	Write the integration with the values given within the problem.
	Use the GDC to calculate the definite integral. 
$= 1.38$	Write down the answer.

The GDC also provides the opportunity for highly original and sometimes very unexpected solutions.

Example question 2

The continuous random variable X has probability density function:

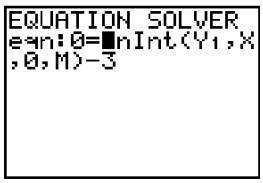
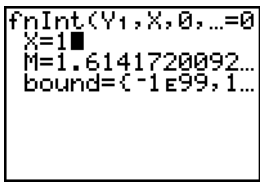
$f(x) = \frac{1}{6}x(1+x^2)$ for $0 \le x \le 2$,

$f(x) = 0$ otherwise.

Find the median of X.

Write down	Rationale
The median m satisfies $\frac{1}{6}\int_0^m (x+x^3)\mathrm{d}x = \frac{1}{2}$	Write down an appropriate mathematical formula representing the median.
$\frac{m^2}{2} + \frac{m^4}{4} = 3$ $\Rightarrow m^4 + 2m^2 - 12 = 0$ $m^2 = \frac{-2 \pm \sqrt{4+48}}{2} = 2.60555...$	One method would be to evaluate the integral and solve it algebraically.
$m = 1.61$	Write down the answer.

An alternative is to use the equation solver. Note that the first step of writing down a mathematical formula does not change, even when using the GDC for the majority of working.

Write down	Rationale
The median m satisfies $\frac{1}{6}\int_0^m (x+x^3)\mathrm{d}x = \frac{1}{2}$	Write down an appropriate mathematical formula representing the median.
$\int_0^m (x+x^3)\mathrm{d}x - 3 = 0$	Simplify and rewrite equal to zero.
	Enter $x + x^3$ as Y1. Then enter the expression into the equation solver (shown below on the TI-84+SE). 
	Highlight and solve for M. 
$m = 1.61$	Write down the answer.

Some "GDC Teaching Moments"

This section focuses on the use of the TI-84+SE and Casio 9850+ calculators when teaching the mathematics HL/SL core syllabus. Four simple teaching examples are presented below, with calculator instructions. They are intended to help teachers who have limited experience of using GDCs to appreciate the GDC as a teaching tool. Teachers should refer to the manufacturers' web sites for additional GDC material and tutorials. Third-party web sites also provide a large amount of GDC material and teaching examples.

Using the GDC in the classroom

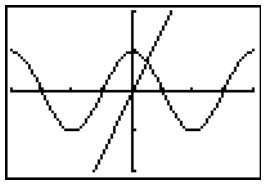
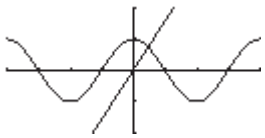
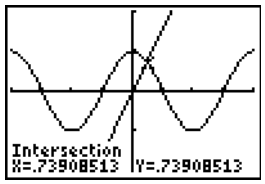
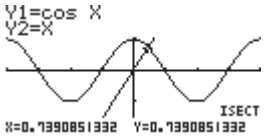
The GDC is most often used in the mathematics classroom to draw graphs and to analyse functions. Occasionally, it is instructive to tackle problems that lack straightforward algebraic solutions, taking advantage of some of the alternative teaching approaches that are possible using GDCs. An example is the "solve" question below, where many approaches could be used.

Solve for x: $\cos x = x$.

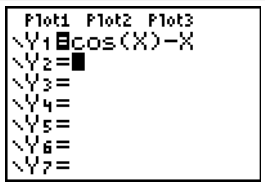
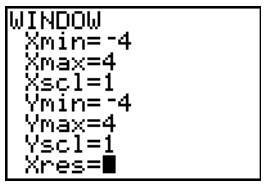
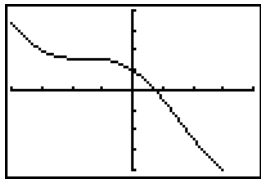
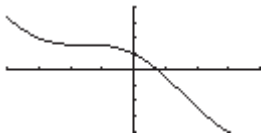
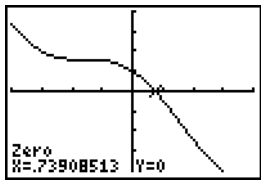
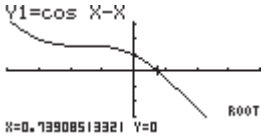
Examples of using the GDC

Task A: Let Y1 represent the expression on the left and let Y2 represent the expression on the right. Graph both and look for intersection(s).

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs. cos X,T,θ,n) ENTER X,T,θ,n ENTER	Plot1 Plot2 Plot3 \Y1■cos(X) \Y2■X \Y3= \Y4= \Y5= \Y6= \Y7=	MENU 5 Remove any stored graphs. cos X,θ,T EXE X,θ,T EXE	Graph Func :Y= Y1■cos X Y2■X Y3: Y4: Y5: Y6: SEL DEL TYPE ZMEM DRAW
2.	Change the window settings to: Xmin = –2π Xmax = 2π Xscl = π/2 Ymin = –2 Ymax = 2 Yscl = 1.		Change the window settings to: Xmin : –2π max : 2π scale : π/2 Ymin : –2 max : 2 scale : 1.	
	WINDOW (–) 2 2nd ^ ENTER 2 2nd ^ ENTER 2nd ^ ÷ 2 ENTER (–) 2 ENTER 2 ENTER 1 ENTER	WINDOW Xmin=-6.283185… Xmax=6.2831853… Xscl=1.5707963… Ymin=-2 Ymax=2 Yscl=1 Xres=■	SHIFT F3 (–) 2 SHIFT EXP EXE 2 SHIFT EXP EXE SHIFT EXP ÷ 2 EXE (–) 2 EXE 2 EXE 1 EXE	View Window Xmin :-6.2831853 max :6.2831853 scale:1.57079632 Ymin :-2 max :2 scale:1 INIT TRIG STD STO RCL

3.	GRAPH		EXE F6	
4.	2nd TRACE 5 ENTER ENTER ENTER		SHIFT F5 F5	

Task B: Rewrite the equation, $\cos x = x$, letting Y1 represent the difference, $\cos x - x = 0$. Graph the Y1 and look for zeros

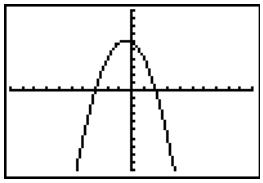
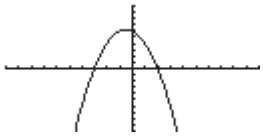
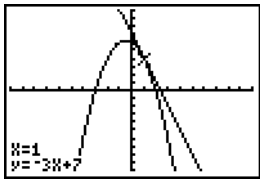
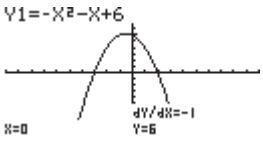
	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs. cos X,T,θ,n) − X,T,θ,n ENTER		MENU 5 Remove any stored graphs. cos X,θ,T − X,θ,T EXE	
2.	Change the window settings to: Xmin = −4 Xmax = 4 Xscl = 1 Ymin = −4 Ymax = 4 Yscl = 1.		Change the window settings to: Xmin : −4 max : 4 scale : 1 Ymin : −4 max : 4 scale : 1.	
	WINDOW (−) 4 ENTER 4 ENTER 1 ENTER (−) 4 ENTER 4 ENTER 1 ENTER		SHIFT F3 (−) 4 EXE 4 EXE 1 EXE (−) 4 EXE 4 EXE 1 EXE	
3.	GRAPH		EXE F6	
4.	2nd TRACE 2 Ensure that the cursor is to the left of the root, then ENTER. › until the cursor is to the right of the root then ENTER. ENTER again when the cursor is close to the root and the prompt asks for a guess.		SHIFT F5 F1	

Task C: Use the solver to solve $\cos x - x = 0$

	TI-84+SE		Casio 9850+	
	Press	**Result**	**Press**	**Result**
1.	MATH 0 ▼ CLEAR	EQUATION SOLVER eqn:0=■	MENU 1 OPTN F4 F1	Solve(Solve d/dx ∫dx/dx ∫ dx
2.	cos X,T,θ,n) − X,T,θ,n ENTER ALPHA ENTER	cos(X)−X=0 X=■7390851321… bound={-1E99,1…	cos X,θ,T − X,θ,T , • 5 , 0 , 1) EXE	Solve(cos X−X,0.5,0,1) 0.7390851332 Solve d/dx ∫dx/dx ∫ dx

Task D: Find the equation of the tangent line to $f(x) = -x^2 - x + 6$ at $x = 1$

Although a GDC is often regarded as a "graphing calculator", its functionality extends well beyond routine graphing applications. For example, a built-in function allows students to find and graph the equations of tangent lines, and some models will even give the equation.

	TI-84+SE		Casio 9850+	
	Use default graph window ± 10 **.**		**Use standard graph window** ± 10 **.**	
	Press	**Result**	**Press**	**Result**
1.	Y= Remove any stored graphs. (−) X,T,θ,n x^2 − X,T,θ,n + 6 ENTER	Plot1 Plot2 Plot3 \Y1■-X²-X+6 \Y2=■ \Y3= \Y4= \Y5= \Y6= \Y7=	MENU 5 Remove any stored graphs. (−) X,θ,T x^2 − X,θ,T + 6 EXE	Graph Func :Y= Y1■-X²-X+6 Y2: Y3: Y4: Y5: Y6: SEL DEL TYPE ZMEM DRAW
2.	GRAPH		F6	
3.	2nd PRGM 5 1 ENTER		sin F4 F2	
	The equation of the tangent line is $y = -3x + 7$. By entering a new value and then ENTER new values of x can be selected and the new tangent line (and equation of the tangent line) will appear.		The Casio 9850+ will only give the gradient for a given value of x.	

Note that the list, matrix and statistical features of the GDC, used singly or in conjunction with each other, provide potential for additional teaching examples.

Managing GDC Memory

GDCs are sophisticated devices with inner workings not very different to those of a personal computer (PC). A GDC has hardware and software running different levels of programming. Teachers should be able to manage the memory and functionality of the GDC used in the classroom and in examinations.

A GDC is capable of displaying tabular, matrix, list, geometrical and graphical representations, in addition to having all the functions found on scientific calculators. GDCs can also run programs and flash ROM applications (Apps).

Not all programs or Apps are suitable for use in examinations (see the list in the *Vade Mecum*). Not all GDCs are suitable for use in examinations that require GDCs (mathematical studies SL, mathematics SL, mathematics HL and further mathematics SL).

All students in the Diploma Programme will require a calculator for one or more subjects. It is **strongly recommended** that students have access throughout the subject course(s) to a single GDC from the list of recommended models (see the list in the *Vade Mecum*).

Basic terminology related to the GDC

This section contains descriptions of key terms and information to help teachers manage the memory and functionality of a typical GDC. Understanding these terms will be of great help in successfully managing GDC use in examinations. While models vary, what is possible on a GDC relies on the hardware, the operating system (OS) and the application programs available.

Hardware

The GDC hardware runs the instructions contained in the code of the OS and other programs in the memory of the calculator. The central processing unit (CPU), along with the amount (and type) of available memory, determines what kind of code can be run and how fast.

Operating system

A GDC comes with an OS developed by the manufacturer specifically for that model. What is often referred to as the "functionality" of a GDC is based on instructions contained in the code of the OS. All this is located in the main memory of the calculator. The functionality of the OS is limited by the hardware and the size (and type) of memory of the calculator. Using the calculator does not change the OS code, but it is possible to replace (or update) the entire OS. Some manufacturers provide OS upgrades via the GDC–computer link, and third-party OS are available for some models. Changing the OS may significantly change the functionality of the calculator. Third-party OS may permanently damage the hardware of the calculator.

Verifying the version of the operating system

	Texas Instruments		Casio
	Go to About Memory menu. 2 + 1 Refer to the manual for older models.		OS is generally fixed and cannot be altered.

Memory

GDCs have different areas within their hardware to store data and code. The available memory on GDCs has recently become quite large, permitting the expansion of the OS and the ability to store a number of different types of programs and data.

- Main memory: The main memory contains the OS code and is not managed by the user, apart from loading alternative OS versions.
- Random Access Memory (RAM): Computations, lists, variables, data and programs can be stored in the RAM memory.
- Archive (Flash Read Only Memory (ROM)): Variables, programs, Apps, and groups can be stored in the archive memory. Not all GDCs have ROM memory.

Functionality, programs and flash applications

The relatively recent introduction of different storage areas and types of code that can be run on a GDC has understandably resulted in some confusion about what a "program" is. The type of programs on a GDC, and their level of sophistication, varies greatly, but can generally be classified by what data the program makes use of, where/how it is stored in the memory and the method that is used to generate/create it. Programs are widely available through the Internet and can be shared using GDC–computer links or GDC–GDC links.

Hardwired programs

The OS code itself is technically a set of programs located within the main memory. The OS, and any other programs contained in the main memory, are often referred to as "hardwired programs" (even though this distinction is technically incorrect). For example, the "program" Finance appears in the Apps menu of the TI-84+SE but cannot be removed using any type of reset. This is because Finance is actually part of the hardwired OS and is not an Apps. TI simply ran out of buttons and needed to create a place from which the program could be easily accessed.

RAM programs

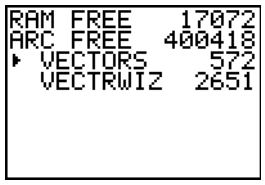
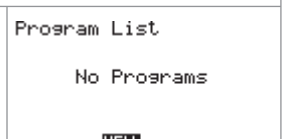
A program stored in RAM is referred to as a "RAM program". These programs are written in one of two programming languages: "Basic" or "Assembler". RAM programs are most often student-generated and are typically keyed directly into the calculator's program editor, which uses a simplified version of the "Basic" programming language.

More commonly, the RAM programs found on the GDC are written on a computer using the "Assembler" programming language and can be shared and downloaded from the Internet using a GDC–computer link. These programs can change the functionality of the GDC.

RAM programs run from the PRGM button on a TI calculator or from the menu icon 

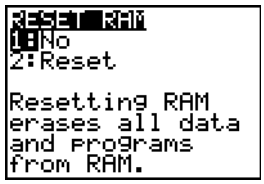
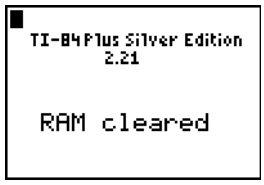
on a Casio calculator. The names of these programs should appear when verifying the RAM memory. A RAM program name can be modified, therefore, the name of a program that is displayed does not necessarily confirm its functionality. It is sometimes necessary to run the program to check its actual functionality.

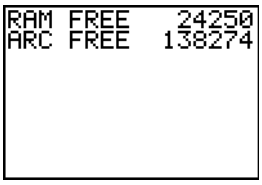
Verifying the RAM memory

	Texas Instruments		Casio	
	Reset Defaults will give the default window, which includes memory usage and programs list.		On a Casio, Memory Usage will list programs and data stored in the calculator.	
	Press	**Result**	**Press**	**Result**
1.	2nd + 2 7		MENU ALPHA tan EXE to access the Memory Usage screen.	
2.	This is a very similar task on almost any TI GDC. You may find you have to search in a number of menus to find the memory information required.		Use Program to list any programs loaded on the Casio.	
			MENU ALPHA log EXE to access the Program List screen.	

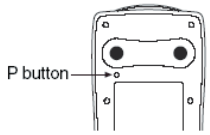
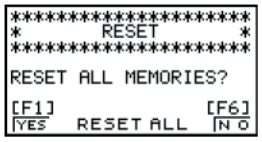
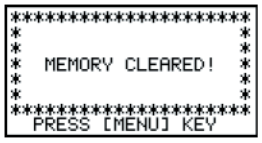
Resetting the RAM portion of the memory removes all RAM programs.

Resetting the RAM memory of the TI-83/84 family of models

	Press	Result
1.	2nd then + to access memory functions.	
2.	7 to access Reset function.	
3.	1 to reset all RAM memory.	
4.	2 to confirm Reset RAM.	

5.	2nd + 2 7 will display full RAM and no files. RAM FREE **24250** TI-84+SE RAM FREE 24289 TI-83+SE RAM FREE 24303 TI-83+	
or	PRGM will display the screen with no programs listed.	

Resetting the RAM memory of the Casio CFX-9850 Plus/FX 9750 Plus/ CFX 9950 Plus/CFX1.0 Plus/Graph 65 Plus/Graph 35 Plus

	Press	Result
1.	Locate the Reset button on the back of the calculator.	
2.	Use a thin, pointed object to press the Reset button. The Reset confirmation screen will appear on the display.	
3.	F1	

If the display appears to be darker or dimmer after you reset the calculator, adjust the colour contrast. The model shown above is the CFX-9850+. See the manufacturer's manual for instructions for other models.

Flash (ROM) applications (Apps)

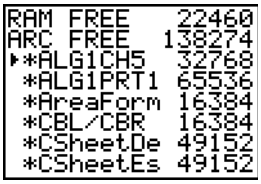
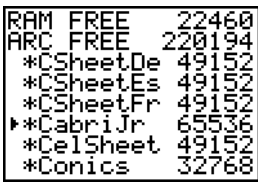
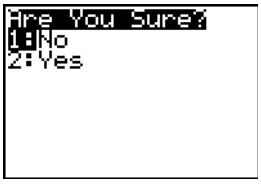
"Flash (ROM) applications" (Apps) are sophisticated programs written by expert programmers. They are stored in the flash ROM memory of the GDC. Many Apps are updates of previously common RAM "Assembler" programs and hence have similar names and functionalities to these older programs. (Some of these older programs, such as Solver, have now been incorporated into the OS.) Most Apps run/install from the Apps menu, which can be accessed by pressing the APPS button on a TI calculator. After they are installed, some Apps change the functionality of the buttons and are run by special sets of button sequences. For example, once installed, the Apps "Catalog Help" is run by pressing + when a function is selected on screen.

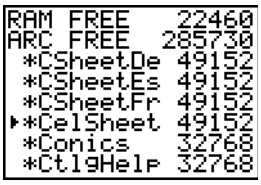
The TI-84+SE is packaged and loaded with a number of Apps. Packaging includes a GDC–computer USB link cable, a CD-Rom containing GDC-link software and backups of all the Apps. Hundreds of additional Apps can be downloaded from the Internet using the GDC–computer link and installed on the GDC by pressing one or two buttons.

Most Casio calculators do not have ROM. One model that does have flash ROM is the Casio FX1.0 Plus. Casio flash ROM applications are called "add-ons" and are available as downloads from one of the Casio support web sites. Installing the add-on introduces a related menu icon on the main menu screen, which can then be selected to run the program.

Resetting the flash (ROM) memory of a TI calculator removes all Apps except "Finance". Finance, as noted above, is a hardwired program but appears in the Apps menu for users' convenience. Casio models must be initialized to remove the add-ons.

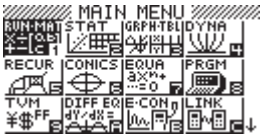
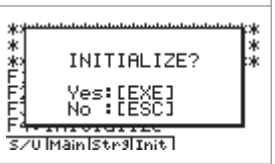
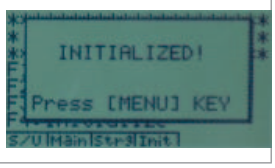
Removing Apps from the ROM memory of the TI-83 Plus/TI-83+SE/ TI-84+/TI-84+SE

	Press	Result
1.	2nd then + to access memory functions.	
2.	2 to access memory management functions.	
3.	ALPHA then MATH to view Apps in calculator memory. (The Apps displayed on this screen may be different to those found on your calculator.)	
4.	▲ and ▼ to scroll up and down to select the Apps to be removed. Arrow indicates selection; on this screen, the Apps CabriJr has been selected.	
5.	DEL to delete the Apps selected. A confirmation screen will appear.	

6.	2 to confirm delete. Updated display of Apps in memory will appear after short pause. The Apps deleted will no longer appear. Repeat until only approved Apps remain.	
7.	APPS to check Apps available on calculator.	

(Note that Finance will not appear in the memory screens, but will only appear when APPS is pressed.)

Initializing the RAM and ROM memories of the Casio FX1.0 Plus

	Press	Result
1.	MENU	
2.	Use the arrow keys to highlight SYSTEM.	
3.	EXE to activate the System Manager.	
4.	F4	
5.	F5 to activate Initialize. Initialize confirm will appear in display.	
6.	EXE to confirm Initialize.	

If the display appears to be darker or dimmer after you reset the calculator, adjust the colour contrast.